Statement of Expenditure and Utilization Certificate
Period-1/06/2015-30/06/2018
(STATISTICS)

Major Research Project
entitled

DEVELOPMENT OF SOME NONPARAMETRIC QUALITY CONTROL CHARTS

(43-542/2014(SR), Date: 30/10/2015)

supported
by

University Grants Commission, New Delhi

submitted
by

Dr. D. T. Shirke
Principal Investigator
Department of Statistics
Shivaji University, Kolhapur 416 004.

Annexure - III

UNIVERSITY GRANTS COMMISSION
BAHADUR SHAH ZAFAR MARG
NEW DELHI – 110 002
STATEMENT OF EXPENDITURE IN RESPECT OF MAJOR RESEARCH PROJECT

1. Name of Principal Investigator: Digambar Tukaram Shirke
2. Deptt. of Principal Investigator: Department of Statistics,
   University/College- Shivaji University, Kolhapur.
3. UGC approval Letter No. 43-542/2014(SR) and Date: 30/10/2015
4. Title of the Research Project: Development of Some Nonparametric Quality Control Charts
5. Effective date of starting the project: 01/07/2015
6. a. Period of Expenditure: From 01/06/2015 to 30/06/2018

   b. Details of Expenditure: Rs. 10,25,043/- (Rupees ten lakh twenty five thousand and forty three only) out of Rs. 11,19,193/- (Rupees eleven lakh nineteen thousand and one hundred ninety three only)

S.No.	Item	Amount Approved (Rs.)	Total Amount Received	Expenditure Incurred (Rs.)				Total Expenditure (Rs.)	Balance
				2015-16	2016-17	2017-18	April-June 2018		
i.	Books & Journals	1,00,000/-	1,00,000/-	0/-	1,00,000/-	0/-	0/-	1,00,000/-	0/-
ii.	Equipment	3,00,000/-	3,00,000/-	1,68,385/-	56,135/-	0/-	0/-	2,24,520/-	75,480/-
iii.	Project Fellow (Including HRA)	5,76,000/-	4,76,593/-	47,419/-	1,54,000/-	2,66,891/-	57,600/-	5,25,910/-	-49,317/-
iv.	Contingency	30,000/- (20% re-appropriated from Field Work/Travel and Hiring Services)	27,000/-	230/-	4,875/-	9,566/-	8,605/-	23,276/-	3,724/-
v.	Field Work/Travel (Details given in the proforma at Annexure- IV).	1,25,000/-	1,17,000/-	5,802/-	45,256/-	17,654/-	4,025/-	72,737/-	4,4263/-
vi.	Hiring Services	45,000/-	36000/-	2,000/-	2,500/-	0/-	11,500/-	16,000/-	20,000/-
vii.	Chemicals & Glassware	0/-	0/	0/-	0/-	0/-	0/-	0/-	0/-
viii.	Overhead	62,600/-	62,600/-	0/-	31,300/-	31,300/-	0/-	62,600/-	0/-
ix.	Any other items (Please specify)	0/-	0/-	0/-	0/-	0/-	0/-	0/-	0/-
	Total	12,38,600/-	**11,19,193/-**	2,23,836/-	3,94,066/-	3,25,411/-	81,730/-	**10,25,043/-**	**94,150/-**

c . Staff

Date of Appointment: 19/12/2015

S.No	Items	Amount Approved (Rs.)	From	To	Expenditure Incurred (Rs.)
1.	**Project fellow:** Non-GATE/Non-NET- Rs. 14,000/- p.m. p.m. for initial 2 years and Rs. 16,000/- p.m. for the third year. (Including HRA)	5,76,000/-	19/12/2015	31/03/2016	47,419/-
			01/04/2016	28/02/2017	1,54,000/-
			19/12/2015	28/02/2017 (HRA)	40,284/-
			01/03/2017	31/03/2018	2,26,607/-
			01/04/2018	30/06/2018	57,600/-
		5,76,000/-	Total		5,25,910/-

1. It is certified that the appointment(s) have been made in accordance with the terms and conditions laid down by the Commission.
2. If as a result of check or audit objection some irregularly is noticed at later date, action will be taken to refund, adjust or regularize the objected amounts.
3. Payment @ revised rates shall be made with arrears on the availability of additional funds.
4. It is certified that the grant of Rs. 11,19,193/- (Rupees eleven lakh nineteen thousand and one hundred ninety three only) received from the University Grants Commission under the scheme of support for Major Research Project entitled "Development of Some Nonparametric Quality Control Charts" vide UGC letter No. F. 43-542/2014(SR) dated 30/10/2015. Out of the received grant Rs. 10,25,043/- (Rupees ten lakh twenty five thousand and forty three only) [Rs. 2,23,836/- for the period 01/07/2015 to 31/03/2016, Rs. 3,94,066/- for the period 01/04/2016 to 31/03/2017, Rs. 3,25,411/- for the period 01/04/2017 to 31/03/2018 and Rs. 81,730/- for the period 01/04/2018 to 30/06/2018] has been utilized for the purpose for which it was sanctioned and in accordance with the terms and conditions laid down by the University Grants Commission.

SIGNATURE OF THE PRINCIPAL INVESTIGATOR

REGISTRAR (Seal)

Registrar
Shivaji University, Kolhapur

STATUTORY AUDITOR (Seal)

Sankpal Kulkarni & Associates
Chartered Accountants

(Shrirang Kulkarni)
Partner M No. 108722

Sankpal Kulkarni & Associates Chartered Accountants Firm No. 121256W

Annexure - I

UNIVERSITY GRANTS COMMISSION
BAHADUR SHAH ZAFAR MARG
NEW DELHI – 110 002
STATEMENT OF EXPENDITURE INCURRED ON FIELD WORK

Name of the Principal Investigator: Dr. D. T. Shirke

Name of the Place visited	Duration of the Visit		Mode of Journey	Expenditure Incurred (Rs.)
Year 2015-16				
1. Statistical Quality control & Research Unit	06/11/2015	06/11/2015	By rental car	4,982/-
2. Department of Statistics, Shivaji University, Kolhapur. Visit of Dr. V. B. Ghute under project fellow selection committee	11/12/2015	11/12/2015	By bus	820/-
Year 2016-17				
3.Savitribai Phule University, Pune.	24/03/2016	25/03/2016	Own car	4,717/-
4. Department of Statistics, Shivaji University, Kolhapur.Visit of Dr. S. K. Khilare for project related work	30/05/2016	01/06/2016	By Bus	1,490/-
5. Savitribai Phule Pune University, Pune	07/05/2016	07/05/2016	By rental car	4,902/-
6. Department of Statistics, Mumbai University, Mumbai	29/06/2016	29/06/2016	By rental car	7,942/-
7.Department of Statistics, Mumbai University, Mumbai and IIT Bombay , Mumbai	13/11/2016	17/11/2016	By rental car	11,780/-
8.UGC office Delhi, Midterm Review Workshop	28/02/2017	01/03/2017	By Air+ rental car	14,425/-
Year 2017-18				
9. Savitribai Phule Pune University, Pune	13/05/2017	13/05/2017	By own car	4,515/-
10. Bangalore University	13/03/2018	16/03/2018	By rental car	13,139
April-June 2018				
11. Savitribai Phule Pune University, Pune	12/05/2018	12/05/2018	Own car	4,025/-
			Total	72,773/-

Certified that the above expenditure is in accordance with the UGC norms for Major Research Projects.

SIGNATURE OF PRINCIPAL INVESTIGATOR

REGISTRAR/PRINCIPAL
(Seal)
Registrar
Shivaji University, Kolhapur

Annexure - V

UNIVERSITY GRANTS COMMISSION

BAHADUR SHAH ZAFAR MARG

NEW DELHI – 110 002

Utilization certificate

Certified that the grant of Rs. 11,19,193/- (Rupees eleven lakh nineteen thousand and one hundred ninety three only) received from the University Grants Commission under the scheme of support for Major Research Project entitled "Development of Some Nonparametric Quality Control Charts" vide UGC letter No. F. 43-542/2014(SR) dated 30/10/2015. Out of the received grant Rs. 10,25,043/- (Rupees ten lakh twenty five thousand and forty three only) [Rs. 2,23,836/- for the period 01/07/2015 to 31/03/2016, Rs. 3,94,066/- for the period 01/04/2016 to 31/03/2017, Rs. 3,25,411/- for the period 01/04/2017 to 31/03/2018 and Rs. 81,730/- for the period 01/04/2018 to 30/06/2018] has been utilized for the purpose for which it was sanctioned and in accordance with the terms and conditions laid down by the University Grants Commission.

SIGNATURE OF THE PRINCIPAL INVESTIGATOR	REGISTRAR (Seal)	STATUTORY AUDITOR (Seal)

Registrar
Shivaji University, Kolhapu.

Sankpal Kulkarni & Associates
Chartered Accountants

(Shrirang Kulkarni)
Partner M No. 108722

Sankpal Kulkarni & Associates Chartered Accountants Firm No. 121256W

Month-Wise detailed statement of expenditure towards salary and HRA of project fellow

Name of Project Fellow: Mahesh Shivaji Barale
Qualification: M.Sc. (Statistics)
Date of appointment: 19/12/2015

Note: HRA @ 20% (Rs. 2,800/-) of salary as per University Rules for 1st two years and (Rs. 3200) for 3rd year.

S.No	From	To	Expenditure Incurred (Rs.)	HRA (Rs.)
1	19/12/2015	31/12/2015	5,419/-	-
2	01/01/2016	31/01/2016	14,000/-	-
3	01/02/2016	29/02/2016	14,000/-	-
4	01/03/2016	31/03/2016	14,000/-	-
5	01/04/2016	30/04/2016	14,000/-	-
6	01/05/2016	31/05/2016	14,000/-	-
7	01/06/2016	30/06/2016	14,000/-	-
8	01/07/2016	31/07/2016	14,000/-	-
9	01/08/2016	31/08/2016	14,000/-	-
10	01/09/2016	30/09/2016	14,000/-	-
11	01/10/2016	31/10/2016	14,000/-	-
12	01/11/2016	30/11/2016	14,000/-	-
13	01/12/2016	31/12/2016	14,000/-	-
14	01/01/2017	31/01/2017	14,000/-	-
15	01/02/2017	28/02/2017	14,000/-	-
16	19/12/2015	28/02/2017	-	40,284/-
17	01/03/2017	30/04/2017	28,000/-	5,600/-
18	01/05/2017	31/10/2017	84,000/-	16,800/-
19	01/11/2017	30/11/2017	14,000/-	2,800/-
20	01/12/2017	31/12/2017 (3rd year start 19th Dec)	14,839/-	2,968/-
21	01/01/2018	28/02/2018	32,000/-	6,400/-
22	01/03/2017	31/03/2018	16,000/-	3,200/-
23	01/04/2018	30/06/2018	48,000/-	9,600/-
		Total	4,38,258/-	87,652/-
			Total Salary with HRA	**5,25,910/-**

Principal Investigator

Annexure -VIII

UNIVERSITY GRANTS COMMISSION

BAHADUR SHAH ZAFAR MARG

NEW DELHI – 110 002.

Annual Report of the work done on the Major Research Project.

1. Project report No. – Final
2. UGC Reference No.F. - 43-542/2014(SR)
3. Period of report: from - 01/04/2018 to 30/06/2018
4. Title of research project - Development of Some Nonparametric Quality Control Charts
5. (a) Name of the Principal Investigator- Dr. D. T. Shirke

   (b) Deptt. – Department of Statistics

   (c) University where work has progressed- Shivaji University, Kolhapur
6. Effective date of starting of the project - 01/07/2015
7. Grant approved and expenditure incurred during the period of the report:

   a. Total amount approved Rs. 12,38,600/-

   b. Total expenditure Rs. 10,25,043/-

   c. Report of the work done: Please see Annexure-I

   i. Brief objective of the project

   a) To take review of the existing nonparametric control charts.
   b) To propose some new nonparametric control charting procedures using signed-rank tests, generalised signed-rank tests, concept of data depth etc.
   c) To investigate the performance of the nonparametric control charts through various distributional quantities like average run length, percentiles for various continuous process distributions like Normal, Cauchy, Double exponential etc.
   d) To develop computer programs for designing and for comparing control charts with the existing parametric/distribution-free control charts.
   e) To investigate merits and demerits of the nonparametric control chart and recommend areas for applications.

   ii. Work done so far and results achieved and publications, if any, resulting from the work (Give details of the papers and names of the journals in which it has been published or accepted for publication)- Details of work done so far is given in Annexure-I.

   iii. Has the progress been according to original plan of work and towards achieving the objective. if not, state reasons

   iv. Please indicate the difficulties, if any, experienced in implementing the project: Nil

v. If project has not been completed, please indicate the approximate time by which it is likely to be completed. A summary of the work done for the period (Annual basis) may please be sent to the Commission on a separate sheet- NA

vi. If the project has been completed, please enclose a summary of the findings of the study. One bound copy of the final report of work done may also be sent to University Grants Commission- NA

vii. Any other information which would help in evaluation of work done on the project. At the completion of the project, the first report should indicate the output, such as

(a) Manpower trained (b) Ph. D. awarded (c) Publication of results (d) other impact, if any- A student has registered for Ph.D. in 2017-2018 with specialisation statistical quality control.

SIGNATURE OF THE PRINCIPAL INVESTIGATOR

REGISTRAR/PRINCIPAL

(Seal)

Registrar
Shivaji University, Kolhapu.

Annexure – IX

UNIVERSITY GRANTS COMMISSION
BAHADUR SHAH ZAFAR MARG
NEW DELHI – 110 002
PROFORMA FOR SUBMISSION OF INFORMATION AT THE TIME OF SENDING THE
FINAL REPORT OF THE WORK DONE ON THE PROJECT

1. Title of the Project: Development of Some Nonparametric Quality Control Charts
2. Name and Address of the Principal Investigator: Dr. D. T. Shirke
3. Name and Address of the Institution: Department of Statistics, Shivaji University, Kolhapur, Maharashtra PIN-416004.
4. UGC Approval Letter No.: 43-542/2014(SR), Date:30/10/2015
5. Date of Implementation: 01/07/2015
6. Tenure of the Project: 01/07/2015-30/06/2018
7. Total Grant Allocated: Rs. 12,38,600/-
8. Total Grant Received: Rs. 11,19,193/-
9. Final Expenditure: Rs. 10,25,043/-
10. Title of the Project: Development of Some Nonparametric Quality Control Charts
11. Objectives of the Project
    a) To take review of the existing nonparametric control charts.
    b) To propose some new nonparametric control charting procedures using signed-rank tests, generalised signed-rank tests, concept of data depth etc.
    c) To investigate the performance of the nonparametric control charts through various distributional quantities like average run length, percentiles for various continuous process distributions like Normal, Cauchy, Double exponential etc.
    d) To develop computer programs for designing and for comparing control charts with the existing parametric/distribution-free control charts.
    e) To investigate merits and demerits of the nonparametric control chart and recommend areas for applications.
12. Whether Objectives Were Achieved: Yes, more than 5 control charts are developed.
13. Achievements from the Project: 1 Ph.D. degree is awarded to student with specialisation statistical quality control in 2017-2018.
14. Summary of the Findings:

An extensive review of the existing nonparametric control charts has been taken. Reputed journals were browsed for articles on nonparametric control charts. Further weaknesses and strengths of the nonparametric charts were also reviewed. There are research articles published by researchers working at national and international Universities/Institutes. In the recent years, control charts based on adaptive sampling scheme has received importance as it requires on an average smaller time to detect the shift. In the light of the same, a nonparametric control chart based on variable sampling procedure is proposed and studied for its performance. An article based on this work was submitted to Communications in Statistics, Computations and Simulation for possible publication. The article was reviewed by the referees and the same has been revised in the light of comments received.

Design of control charts is also one of the important aspects for implementation of control charts. Relatively less has been reported on this aspect. We have proposed various economic design schemes for nonparametric control charts. Economic Design of a nonparametric exponentially weighted moving average control Chart for location has been studied and an article based on the same has been published. Economic design of variable sampling interval sign control chart has also been worked out and an article has appeared research journal.

As control charts for monitoring variability, which is based on sign statistic using in-control deciles is proposed. This control chart is studied for its performance. It is observed that the proposed control chart outperforms for various process distributions. The work has been carried out in collaboration with Professor S. Chakraborti, USA. The same was submitted to Quality Engineering Journal. The Cumulative Sum (CUSUM) Control chart is popular for detecting shifts of smaller magnitude. We have constructed CUSM chart based on sign statistic using in-control deciles. The ARL performance of the CUSUM chart is studied for various process distributions. When the process distribution is normal then CUSUM S^2 chart performs better than nonparametric CUSUM chart. An effect of non-normal data on CUSUM S^2 chart is reported. The performance of the proposed control chart is also studied for various distribution for sample size n=10,15,20. Due to the simplified procedure of proposed CUSUM chart use of proposed CUSUM chart is recommended.

A notion of data depth has been used to measure centrality/outlyingness of a given point in a given data cloud. Liu (1995) provided control chart for multivariate process based on data depth. However the performance of the chart has been not reported. Li and Liu (2004) have given a tests for location of multivariate distribution. Using these tests we proposed nonparametric control charts to detect shift in the location of multivariate process. We investigate the performance of the proposed control charts using average run length measure for various continuous distributions. We compare proposed chart with the chart due to Liu (1995). It is observed from extensive simulation studies that the proposed chart performs superior to the chart due to Liu (1995).

15. Contribution to the Society:

The proposed work is related to online process control. Quality of any process or product is directly related to the performance or fitness for use of the end product. Controlling quality of the product and giving the best to the end user is one of the primary objectives of any producer. The work carried out under the project has direct relevance in improving quality of the product. The present methods have certain limitations and are applicable only under specific environment. The reported work provides methods which are useful in improving quality of the process/product under very wide environment.

16. Whether any Ph.D. Enrolled/Produced out of the project: A student has registered for Ph.D. in 2017-2018 with specialisation statistical quality control. 1 Ph.D. degree is awarded to student with specialisation statistical quality control in 2017-2018.

17. No. of Publications out of the Project: 3 articles are published in reputed international journals.
List is attached.

Principal Investigator

Registrar
(Seal)

Registrar
Shivaji University, Kolhapur

Annexure -I

Introduction

Statistical process control is being used to distinguish between two sources of process variation, namely variation due to chance cause and variation due to assignable cause. Chance causes are those, which cannot be economically identified and corrected, while assignable causes are those, which can be identified and can be corrected as well. It is well known that, if a process operates only under chance causes, it is said to be in a state of statistical control (in-control), otherwise it is said to out of control. Use of control charts makes easy to identify and to eliminate assignable causes so that the state of statistical control is ensured. If there is a change in the process then a control chart should detect the same as quickly as possible. It should give an out-of-control signal at the earliest. An efficient control chart detects the change quickly. In order to evaluate performance of any chart, we focus mainly on the number of samples or subgroups that need to be collected before the first out-of-control signal is given by a chart. This is a random variable and is called the run length of the chart. The distribution of run length is helpful in studying properties of the control chart. One of the mostly used measures of chart performance is the expected value of the run length distribution, called the average run length (ARL), and a related characteristic is the variance. Sometimes we use other measures, such as the median or other percentiles. A good chart is the one, which has large ARL when the process is in-control and small ARL when the process is out-of-control. It is also necessary to look into possible errors that a control chart can make. In that case, the probability that a chart signals a process change when in fact there is no change, that is, when the process is in-control is considered, which is a false alarm rate. The false alarm rate is similar to a Type I error in the context of hypothesis testing. We compare two control charts on the basis of out-of-control ARL, such that their respective in-control ARL's are exactly or at least approximately the same.

In literature control charts have been developed with some parametric assumption. That is the characteristics under study are assumed to have some parametric distribution. Most of the times normal distribution is assumed. Therefore such control charts may not perform up to expectation, if the assumption of normality fails to satisfy. There are various situations, where normal distribution may not hold good. One may refer to Shewhart (1939), Ferrell (1953), Tukey (1960), Langenberg and Iglewicz (1986), Jacobs (1990), Alloway and Raghavachari (1991), Yourstone and Zimmer (1992), Woodall and Montgomery (1999), and Woodall (2000) for more details. One may refer to refer to Noble (1951), Tukey (1960), Lehmann (1983), and Gunter (1989) for normal-like but heavier-tailed distributions for more details. These authors and others, including practitioners, provide ample justifications for the development and application of control charts with properties that do not depend on assumption of normality or any other specific parametric distributional assumption.

A formal definition of a nonparametric or distribution-free control chart is given in terms of its in-control run length distribution. If the in-control run length distribution remains the same for every continuous distribution, then the chart is called distribution-free. Therefore nonparametric charts do not require any distributional assumption, while maintaining the in-control ARL at a specified value. Also, the nonparametric charts are likely to share the robustness properties of nonparametric tests and confidence intervals and are, therefore, far more likely to be less impacted by outliers. Nonparametric control charts have the following advantages:

(i) Nonparametric charts are simple to use.
(ii) Nonparametric control charts the lack of a need to assume a particular distribution for the underlying process;

(iii) An in-control run length distribution corresponding to any distribution-free control chart is the same for all continuous distributions. Therefore different nonparametric charts are compared more easily.
(iv) Distribution-free procedures are robust, hence control charts based on distribution-free statistic also have greater robustness and outlier resistance.
(v) It has been reported in the literature that distribution-free control charts are efficient in detecting changes when the true distribution is markedly non-normal, particularly with heavier tails
(vi) Distribution-free control charts do not require, estimate of the variance to set up charts for the location parameter.

It is very clear that nonparametric methods can be somewhat less efficient than their parametric counterparts, which is also the fact for distribution free control charts.

It is required to monitor process location and process dispersion, for detecting the assignable causes in process. The charts R chart, S chart and S^2 chart are usually used to monitor the process variability, which require the assumption of normality. R chart is less efficient than S^2 when the process distribution is normal. Due to non-normality, probability of a signal may appear to be small in comparison with the commonly used probabilities for a Type I error in statistical hypothesis testing. There can be rather large differences between the ARL for non-normal distributions and the ARL calculated under the normality assumption. In literature Das (2008) and proposed a nonparametric control chart for controlling variability based on squared rank test. Amin and Widmaier (1999) proposed sign control charts with variable sampling interval policy for process median and process variability. I proposed a sign chart for variability based on in-control deciles with variable sampling interval policy, which is adaptive version of my nonparametric chart for variability based on deciles.

Multivariate data arise in number of the situations. Most of the multivariate data analysis techniques are based on the assumption of multivariate normality. Justification for multivariate normality is often difficult. In this situation the Hotelling's T^2 chart is widely used to monitor mean vector of the process. Hotelling's T^2 chart proposed by Hotelling's (1947) requires multivariate normality of quality characteristics. In practice, assumption of multivariate normality will not be fulfilled. Therefore implementation of Hotelling's T^2 chart is not correct in such situations. Recently data depth based nonparametric multivariate analysis techniques have found to be more attractive, which do not require normality. A notion of data depth has been used to measure centrality/outlyingness of a given point in a given data cloud. This leads to a natural center outward ranking of multivariate observations. This ordering is found to be very useful in statistical analysis.

Executive Summary of the work done under UGC Major Research Project Report

The work carried out under the project is related to development of nonparametric control charts. An extensive review of the existing nonparametric control charts has been taken. Reputed journals were referred for the latest work reported by other researchers on nonparametric control charts. Further weaknesses and strengths of the existing nonparametric charts were also reviewed.

Nonparametric synthetic and side-sensitive synthetic control charts are proposed using signed-rank statistic to improve the performance of a control chart based on signed-rank statistic. Performance of the proposed control charts measured in terms of average run length for various symmetric distributions. Average run length of the proposed control charts is computed by employing the Markov chains approach. Average run length of the proposed control charts is compared with the signed-rank control chart and it is observed that proposed control charts performs significantly better than the signed-rank control chart. The side-sensitive synthetic control chart also performs significantly better than the two-sided synthetic control chart for all shifts.

A nonparametric control chart is proposed for monitoring process variability, which is based on the known in-control deciles. The chart is motivated from a nonparametric control chart based on in-control quartiles due to Amin et al. (1995). The proposed chart has been studied for its performance for various process distributions. It is simple to use and has attractive out of control ARL performance properties. We illustrate the chart through an example and recommend its use to monitor process variability.

A nonparametric cumulative sum control chart is proposed for process dispersion based on the sign statistic using in-control deciles. The chart can be viewed as modified control chart due to Amin et al. (1995), which is based on in-control quartiles. An average run length performance of the proposed chart is studied using Markov chain approach. An effect of non normality on cumulative sum S^2 chart is studied. The study reveals that the proposed cumulative sum control chart is a better alternative to parametric cumulative sum S^2 chart, when the process distribution is non-normal. We provide an illustration of the proposed cumulative sum control chart.

A nonparametric quality control chart for monitoring process variability based on in-control deciles with variable sampling intervals policy was considered. This chart is motivated from our proposed sign chart based on the deciles and sign chart with variable sampling intervals proposed by Amin et al. (1999). The performance of the proposed chart is studied for various process distributions. The proposed chart is simple to use and has an attractive out of control adjusted average time to signal performance. We have provided implementation of the chart through an example.

A notion of data depth has been used to measure centrality/outlyingness of a given point in a given data cloud. Liu (1995) provided control charts for multivariate process based on data depth. However, the performance of the chart has been not reported. Li and Liu (2004) have given tests for location of multivariate distribution. Using one of these tests we proposed nonparametric control chart to detect shift in the location of multivariate process. We investigate the performance of the proposed control charts using average run length measure for various continuous distributions. We compare proposed chart with the chart due to Liu (1995). It is observed from extensive simulation studies that the proposed chart performs superior to the chart due to Liu (1995).

An economic design of Exponentially Weighted Moving Average control chart based on sign statistic is proposed to control location parameter of the process. The economic performance of the chart is evaluated for different shifts in the location. It is observed that, as shift in the process location increases, sample size to detect the shift and the loss cost from the process decrease. The power of the chart increases with increasing shift. The design gives better economic/statistical performance for large shifts in the process.

An economic design of variable sampling interval (VSI) control chart is proposed based on sign statistic.

- Details of publications resulting from the project work as follows:

1. Economic Design of a Nonparametric EWMA Control Chart for Location, Production, 26(4), 698-706, 2016.
2. Economic design of variable sampling interval sign control chart. Journal of Industrial and Production Engineering, 34(4), 253-260.
3. A nonparametric CUSUM chart for process dispersion. Quality and Reliability Engineering International, 34(5), 858-866.

- Papers submitted for the possible publication:

1. A Nonparametric Control Chart for Monitoring Variability Based on the Deciles
2. A Variable Sampling Interval Sign Chart for Variability Based on Deciles
3. Nonparametric Side-Sensitive Synthetic Control Chart Based On Signed-Rank Statistic
4. Nonparametric control charts based on Data Depth.

Production, 26(4), 698-706, out./dez. 2016
http://dx.doi.org/10.1590/0103-6513.209916

Economic design of a nonparametric EWMA control chart for location

Patil Subhash Hariba[a]*, Shirke Digambar Tukaram[b]
[a]Doodhsakhar Mahavidyalaya, Bidri, Dist-Kolhapur, State-Maharashtra, India
[b]Shivaji University, Kolhapur, Dist-Kolhapur, State-Maharashtra, India
*psubhash_2007@rediffmail.com

Abstract

In this article, we have proposed an economic design of Exponentially Weighted Moving Average control chart based on sign statistic to control location parameter of the process. The economic performance of the chart is evaluated for different shifts in the location. It is observed that, as shift in the process location increases, sample size to detect the shift and the loss cost from the process decrease. The power of the chart increases with increasing shift. The design gives better economic/statistical performance for large shifts in the process. This economic procedure can be applied to any process having known or unknown process outcome distribution. The sensitivity of the design is also carried out to check the effect on statistical as well as economic performance of the design due to change in different time and cost parameters.

Keywords

Economic design. Production cycle. EWMA control chart. Markov chain. Expected loss.

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1. Introduction

Control charts are commonly used as a statistical tool for online process control, to maintain the measurement of quality characteristics of the product produced in between certain limits known as upper control limit (UCL) and lower control limit (LCL). The target value of the process location is set and is referred to as centre line (CL). This tool is first developed by Shewhart (1931). Since then, it is used worldwide for the control of statistical as well as economic performance of the process. In statistical control charts, the overall concentration is made to maintain the statistical constants of the chart such as type I error probability (α), power (1-β) of the chart etc. whereas, in the economic design of control chart, process is targeted to minimize overall loss from the process so that, process can earn maximum profit. In the literature, there are so many types of control charts such as, mean and range ($\overline{X}$ and R) charts, moving average charts, exponentially weighted moving average (EWMA) charts, cumulative sum (CUSUM) charts etc. There are some charts designed also for attribute data which are, control chart for number of defectives (d chart), control charts for number of defects (c chart), control charts for fraction defectives (np chart) etc. Also, there are some non-parametric charts such as sign chart, signed rank chart etc. All these charts have their own statistical properties.

In the economic design of control charts, an expression for the loss (gain) per unit time or per unit produced is obtained, and is optimized with respect to the design parameters; sample size (n), sampling interval (h) and control limit multiplier (k) in terms of sigma units. Duncan (1956) first introduced an economic design for $\overline{X}$ control chart by applying numerical method to obtain optimal values of design parameters that optimizes the loss cost from the process design. Since then, considerable work on economic design of control charts is carried

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out; Montgomery (1980) has reported a review of it up to 1980. Rahim (1985, 1989) has obtained an economic design of $\overline{X}$ chart under non-normality and obtained optimal design parameters for $\overline{X}$ and R chart. Lorenzen & Vance (1986) proposed unified approach for economic design of control charts. Saniga (1989) has obtained economic-statistical design of $\overline{X}$ and R chart, which gives economic design with improved statistical properties at slight increase in loss cost. The considerable work on economic design of control charts is also carried out by Banerjee & Rahim (1987), Reynolds Junior et al. (1988), McWillims (1989), Koo & Case (1990), Bai & Lee (1998), Chou et al. (2001), Chen (2004), Chen & Yeh (2006), Mahadik & Shirke (2007), Patil & Rattihalli (2009), Yeh & Chen (2010), Patil & Shirke (2015) and many others. The work by these researchers has made consistent improvement in the economic design of different types of control chart.

It is observed that, though the CUSUM and/or EWMA charts are effective to detect the small and moderate shifts in the process, less is reported on economic design of these control charts. Further, the practitioners, as they are not aware of these charts, also avoid using these charts. In review of the work on economic design of CUSUM/EWMA chart, we observe; Pan & Chen (2005) have obtained economic design of CUSUM chart for monitoring environmental performance, Serel & Moskowitz (2008) have obtained an economic design of EWMA control chart to monitor process mean and variance jointly using quadratic loss function. Serel (2009) has developed economic EWMA control chart using linear, quadratic and exponential loss function. He et al. (2009) has obtained economic design of EWMA control chart based on Average Run Length (ARL) using response surface methodology (RSM) to search the optimal set of EWMA parameters. Saghaei et al. (2014) have developed economic design of EWMA control chart based on measurement errors using Genetic Algorithm (GA). Chiu (2015) has obtained economic-statistical design of EWMA chart based on quadric loss function using Lorenzen & Vance (1986) approach. Chiu (2015) has used nonlinear programming with statistical constraints to obtain optimum loss per unit time.

Observing this literature, in the present study, an attempt is made to obtain an economic design of EWMA control chart proposed by Yang et al. (2011a) to monitor shifts in the process location. A nonparametric EWMA sign statistic based on signs of observations from the process target value is considered. The nonparametric sign static is used as is independent of process distribution and hence can easily be applied to any control procedure. Only we need to calculate signs of observations from target location. Another advantage is the knowledge of process variance is not required for the implementation of the sign control chart. To reach the purpose, an expression for expected loss cost per unit time is obtained and is minimized with respect to the design parameters of EWMA control chart. An expression for loss cost per unit time is obtained by taking a ratio of the expressions for expected loss cost during a production cycle and expected length of the cycle [Montgomery (1980, 2008)]. The study is aimed to use EWMA chart effectively in economic point of view so that, professionals can use it for any type of process distribution (normal or non-normal). The paper consists of six sections. Section 2, followed by this section describes construction of the EWMA control chart based on sign statistic. The process design is given in section 3. Expressions for expected cycle length and expected loss cost during the production cycle are given in section 4. An example and the sensitivity of the design are carried out in section 5. Conclusions are given in section 6 followed by the references.

2. The EWMA control chart based on sign statistic

Suppose 'X' is a process characteristic of a production process and have a continuous distribution with cumulative distribution function F(.). The process is targeted to control a process location μ having target value μ_0.

Let x_{ij}, i=1, 2, ...; j=1, 2, ... n. be a j^{th} observation from a sub-group sample of size n observed at i^{th} sampling epoch from this process with target location $\mu=\mu_0$.

Define,

$$Y_j = \begin{cases} 1 & ; if\ x_{ij} > \mu_0 \\ 0 & ; otherwise \end{cases} \tag{1}$$

then, $S_i = \Sigma Y_j$, represents one sided sign statistic and gives number of x values in a sample of size n exceeding process target value $\mu=\mu_0$ at i^{th} sampling epoch. Now, S_i becomes a discrete random variable and has binomial distribution with parameters (n, p), where $p=P(x_{ij}> \mu_0)$. If the process location is at median of the process, we have p=1/2 and $E(S_i)=n/2$. If the process location is different from median, value of p will be different than 1/2.

This sign statistic is a slight modification of two sided sign statistic used by Amin et al. (1995) to construct a nonparametric sign control chart. Khilare & Shirke (2010) has also used the sign statistic to develop nonparametric synthetic control chart. The EWMA statistic to monitor small shifts in the process location based on sign statistic is defined as,

$$EWMA_{S_i} = \lambda S_i + (1-\lambda) S_{i-1} \quad (2)$$

where, $0 < \lambda \leq 1$, is a smoothing parameter of EWMA control chart. The starting value of EWMA, denoted as $EWMA_{S_0}$ is taken to be the mean value of S, given by np and have value n/2, if the process location is at median of the process.

The control limits for EWMA chart for long run time according to Yang et al. (2011a) are given by,

$$UCL = np + k\sqrt{\frac{\lambda}{2-\lambda} np(1-p)},$$
$$CL = np, \quad (3)$$
$$LCL = np - k\sqrt{\frac{\lambda}{2-\lambda} np(1-p)},$$

where, k and λ are the properly chosen parameters of EWMA chart, so as to attain certain average run length (ARL), when process is in control.

The ARL of the control chart can be calculated based on Markov Chain approach by Brook & Evans (1972) or by approach by Lucas & Saccucci (1990). To obtain ARL of the chart, the *in-control* region (LCL, UCL) of the control chart is divided into (N-1) sub-intervals of equal width, representing transient states and N^{th} state is an absorbing state. P={pij} be the (N-1)×(N-1) transient probability matrix representing probabilities of moving to state j from state i in one step. These probabilities are calculated using approach by Lucas & Saccucci (1990) and are given by,

p_{ij}= P(moving to state Sj/ being in state Si previously)

=P(EWMA ∈ Sj at time t /EWMA ∈ Si at time (t-1)).

If, **b**= $(b_1, b_2, ..., b_{N-1})'$ is a column matrix of order (N-1) of initial probabilities and **1**= (1, 1, ..., 1)' is a column matrix of order (N-1) having each element equal to one, then the ARL of the control chart is given by,

ARL = **b**'(1-P)**1** (4)

This formula of ARL can be used to find *in-control* and *out-of-control* ARL's of this EWMA control chart. If we use p, as *in-control* probability (say p_0), the ARL computed is *in-control* ARL usually denoted by ARL_0 and when we use p as *out-of-control* probability (say p_1), the ARL computed be *out-of-control* ARL usually denoted by ARL_1. For an illustration, consider EWMA scheme monitored by EWMA chart with parameters λ=0.2 and k=2.84. The *in-control* and *out-of-control* ARL values for this chart are as given in Table 1 and Table 2 respectively for different p values. *Out-of-control* ARL's obtained in Table 2 are for the shift in median.

The ARL's of the chart depends on the values of parameters λ and k of new EWMA control chart. Figure 1 illustrates that, *in-control* ARL increases with increasing values of k and decreasing λ. The same illustration is given by Yang et al. (2011b). Also, they have provided different combinations of λ and k to yield ARL=370 for various n values.

3. Process design

Consider a production process monitored by drawing a sample of size n at an interval of every h hours of production. The process target value is the location parameter of the process quality distribution, when it is in the state of control and is denoted by μ_0. The quality of the product is monitored by a single assignable cause at a time and whenever an assignable cause occurs, the process location (say median) shifts from μ_0 to $\mu_0 + \delta\sigma$, where δ is the shift parameter and σ is the process standard deviation. The process is not self adjusting, that is,

Table 1. ARL_0 values for EWMA control chart.

n\p	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
9	354.99	372.91	379.13	374.07	393.51	387.31	393.51	374.07	379.13	372.91	354.99
10	353.04	340.65	363.59	373.95	375.29	387.71	375.29	373.95	363.59	340.65	353.04
11	352.08	366.74	339.74	351.81	370.20	352.08	370.20	351.81	339.74	366.74	352.08
12	343.16	354.20	382.46	362.66	343.60	376.84	343.60	362.66	382.46	354.20	343.16
13	373.06	363.16	357.54	375.08	377.13	384.24	377.13	375.08	357.54	363.16	373.06
14	376.88	339.85	344.92	363.46	362.23	377.86	362.23	363.46	344.92	339.85	376.88
15	353.28	357.20	371.00	370.23	405.86	363.20	405.86	370.23	371.00	357.20	353.28
16	362.09	357.15	351.34	365.90	387.13	387.90	387.13	365.90	351.34	357.15	362.09
17	357.66	354.98	371.01	366.38	353.59	346.35	353.59	366.38	371.01	354.98	357.66
18	359.23	360.06	348.51	375.69	360.16	393.24	360.16	375.69	348.51	360.06	359.23
19	359.72	365.47	386.07	356.68	353.65	355.82	353.65	356.68	386.07	365.47	359.72
20	331.90	374.76	371.82	358.91	352.12	347.44	352.12	358.91	371.82	374.76	331.90

Table 2. ARL_1 values for EWMA control chart.

n\p	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75
9	5.20	7.27	11.84	25.71	94.32	387.31	94.32	25.71	11.84	7.27	5.20
10	4.80	6.67	10.77	23.22	86.76	387.71	86.76	23.22	10.77	6.67	4.80
11	4.42	6.08	9.64	20.26	74.75	352.08	74.75	20.26	9.64	6.08	4.42
12	4.27	5.82	9.15	19.15	72.40	376.84	72.40	19.15	9.15	5.82	4.27
13	4.07	5.52	8.60	17.84	68.13	384.24	68.13	17.84	8.60	5.52	4.07
14	3.82	5.16	8.02	16.51	63.32	377.86	63.32	16.51	8.02	5.16	3.82
15	3.63	4.90	7.54	15.31	58.14	363.20	58.14	15.31	7.54	4.90	3.63
16	3.55	4.76	7.29	14.73	56.84	387.90	56.84	14.73	7.29	4.76	3.55
17	3.38	4.49	6.78	13.39	50.17	346.35	50.17	13.39	6.78	4.49	3.38
18	3.27	4.36	6.63	13.22	51.25	393.24	51.25	13.22	6.63	4.36	3.27
19	3.17	4.20	6.30	12.34	46.47	355.82	46.47	12.34	6.30	4.20	3.17
20	3.05	4.03	6.02	11.67	43.58	347.44	43.58	11.67	6.02	4.03	3.05

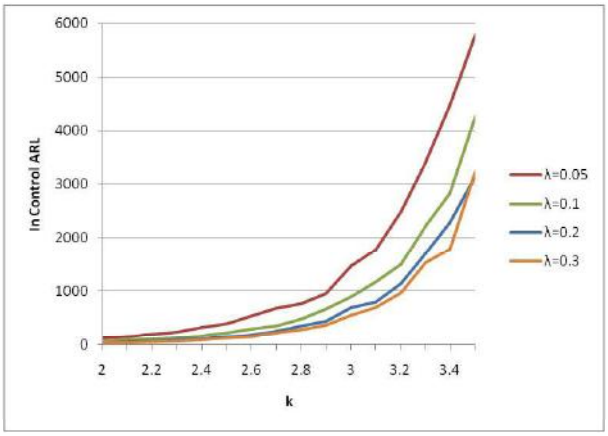

Figure 1. Effect of λ and k on *in-control* ARL (ARL_0).

the process remains in *out-of-control* state, unless and until human interference.

The process is assumed to be start in the state of control. After some random period shift occurs in the process. Consequently, the process signals and will be brought back in control by removing an assignable cause. This period from starting of the process, till drawing back it in the state of control, when an assignable cause occurs in between them is termed to be one production cycle. That is, the time between two successive *in-control* states, given that the shift occurs in between them is known to be one production cycle. After this a fresh cycle begins. For figure of production cycle one can refer Patil & Rattihalli (2009). Further, the transaction between *in-control* and *out-of-control* states is assumed to be instantaneous. In the economic design of control chart, the time required for completing a production cycle and the expected loss during this cycle is determined, which is used to find an expression for expected loss per unit time during a cycle. This expression for expected loss cost is then minimized with respect to design parameters of the control chart to get an economic design.

The notations used in the economic procedure are as below.

n: sample size.

h: sampling interval length in hours.

k, λ: parameters of EWMA control chart scheme.

θ: parameter of the exponential lifetime distribution for *in-control* state.

α: probability of false alarm (probability of Type I error).

β: probability of type II error.

s: expected number of samples taken during *in-control* period.

τ: time of occurrence of the shift in between two consecutive samples.

D: expected search and repair time of an assignable cause during the true alarm.

V_0: per hour income from the process when the process is *in-control*.

V_1: per hour income from the process when the process is *out-of-control*.

C: expected penalty cost per hour due to nonconformities produced while running the process in *out-of-control* state.

E(T): expected length of the production cycle.

E(A): expected income from the process during the production cycle.

E(I): expected income per hour from the process.

ARL_1: *out-of-control* average run length.

ATS: average time to signal.

N: no. of samples taken during the production cycle.

g: time to sample, inspect and conclude one unit in the sample.

a, b: fixed and variable costs of sampling, respectively.

V: loss cost for search of an assignable cause due to single false alarm.

W: loss cost for search and repair of an assignable cause during true alarm.

4. Expected cycle length and loss cost

To obtain an expression for expected loss cost per hour, we have to obtain expressions for expected time period required for a production cycle and the expected loss occurred during that cycle. The expected cycle length consists of an *in-control* period, *out-of-control* period, time for sampling and testing and the time for search and repair of an assignable cause.

The lifetime of the process is always monitored by a failure time (decay) distributions like Exponential distribution (Poisson process) or Weibull distribution, etc. Let us assume that, an assignable cause occurs according to a Poisson process with rate θ, that is, time to occur an assignable cause has an exponential distribution with mean 1/θ. Hence, the expected *in-control* time becomes 1/θ.

Therefore,

$$\textit{In-control}\ \text{Time} = \text{ICT} = 1/\theta \quad (5)$$

Assuming, the shift occurs between i^{th} and $(i+1)^{th}$ sample, the expected time of occurrence of an assignable cause in between i^{th} and $(i+1)^{th}$ sample (τ) according to Montgomery (1980) is,

$$\tau = \frac{\int_{ih}^{(i+1)h}\theta(t-ih)e^{-\theta t}dt}{\int_{ih}^{(i+1)h}\theta e^{-\theta t}dt}, = \frac{e^{\theta h}-(1+\theta h)}{\theta\left(e^{\theta h}-1\right)}. \quad (6)$$

This gives,

$$\text{ATS} = h^{*}\text{ARL}_1 - \tau \quad (7)$$

Therefore, Expected cycle length, which is the time required to complete a cycle is,

$$E(T) = \text{ICT} + \text{ATS} + gn + D,$$

which can be written as,

$$E(T) = 1/\theta + h^{*}\text{ARL}_1 - \tau + gn + D \quad (8)$$

Equation 8 represents, an expression for expected cycle length of a production cycle.

Now, the expected loss cost occurred during a production cycle consist of the loss due to nonconformities produced during the production cycle, loss occurred due to unwanted search of an assignable cause when there is a false alarm, cost for sampling and testing and cost for search and repair of an assignable cause when there is true alarm.

The loss occurred due to nonconformities produced is the difference in the income during *in-control* and *out-of-control* states of the process. Since, V_0 and V_1 are the income per hour from the process, when the process is in *in-control* and *out-of-control* states respectively, providing $C = V_0 - V_1$ as the penalty loss cost due to excess number of nonconformities produced while running the process in *out-of-control* state.

The income from the process during a production cycle is now given by,

$$E(A) = \frac{V_0}{\theta} + V_1\left[h^{*}\text{ARL}_1 - \tau + gn + D\right] \quad (9)$$

which can be written as,

$$E(A) = \frac{V_0}{\theta} + V_0\left[h^{*}\text{ARL}_1 - \tau + gn + D\right] - C\left[h^{*}\text{ARL}_1 - \tau + gn + D\right] \quad (10)$$

This gives expected income per hour as,

$$E(I) = \frac{E(A)}{E(T)} \quad (11)$$

$$= V_0 - \frac{C\left[h^{*}\text{ARL}_1 - \tau + gn + D\right]}{E(T)} \quad (12)$$

Hence, the loss cost per hour due to non-conformities produced (L_1), during a production cycle is given by,

$$L_1 = V_0 - E(I),$$
$$L_1 = \frac{C\left[h^{*}\text{ARL}_1 - \tau + gn + D\right]}{E(T)} \quad (13)$$

The expected number of samples taken during the cycle, E(N) are, $E(N) = \frac{E(T)}{h}$, which gives, the cost for sampling and testing per hour (L_2) as,

$$L_2 = \frac{a+bn}{h} \quad (14)$$

Now, if s denotes expected number of samples taken during *in-control* period and P(i), be the probability that shift occurs during i^{th} and $(i+1)^{th}$ sample, then according to Lorenzen & Vance (1986),

$$P(i) = \int_{ih}^{(i+1)h}\theta e^{-\theta t}dt \quad (15)$$

and,

$$s = \sum_i iP(i), = \sum_{i=0}^{\infty} i\left[e^{-\theta ih} - e^{-\theta(i+1)h}\right], = \frac{e^{-\theta h}}{1-e^{-\theta h}}. \quad (16)$$

Since, α is the probability of false alarm, V is the cost for search of an assignable cause when there is false alarm and W is the cost for search and repair of an assignable cause when there is true alarm, then cost per hour for search and repair of an assignable cause (L_3) is,

$$L_3 = \frac{V\alpha s + W}{E(T)} \quad (17)$$

From Equations 13, 14 and 17, the expected total loss cost per hour during a production cycle is,

$$E(L) = L_1 + L_2 + L_3 \quad (18)$$

Equation 18 represents the equation for the loss cost per unit time from the process during a production cycle. This loss cost function can be minimized with respect to the design parameters of a EWMA control chart, under the condition that, an *out-of-control* probability (p_1) of the process is known. A computer program in *MATLAB* using pattern search method is written for expected loss cost E(L) in Equation 18 and is optimized for different values of n to get optimum design of the EWMA control chart. This program can be operated for different values of design parameters in possible range so as to reach minimum possible loss from the process design. The illustration using real life example is given in the following section 5.

5. An example

To illustrate the design, consider an example of nonreturnable glass bottle production process given by Montgomery (2008). The time and cost parameters used are g=0.0167h, D=1h, C=\$100, a=\$1, b=\$0.1, V=\$50 and W=\$25. The *in-control* run length parameter is θ=0.05.Assuming, process quality distribution is being normal with mean μ and variance 1, optimum design parameters of EWMA control chart (that is, sample size, sampling interval, smoothing parameter and control limit coefficient) to control median of the process are obtained. The optimum design parameters and loss cost occurred are given in Table 3 for different values of shift in the process ranging from 0.5 to 3.0. *Out-of-control* probabilities p are calculated for particular shift and are used for further calculations of ARL_1 values and optimum design parameters of EWMA control chart design.

Table 3 reveals that, as shift in the process goes on increasing, the loss cost from the process goes on decreasing. The sample size is the same for a group of shifts. We note that, for shift equal to 0.5; optimum sampling interval is h=0.6 and sample size is n=12. Also, for shift equal to 1.2 optimum sampling interval is h=1 and sample size is n=12. In the later case, we sample at 1 hour as compared to the former case, where sampling is done at 0.6 hour. But in both the cases optimum sample size is the same. Thus sample size decreases with respect to increase in shift. The parameter λ of the EWMA chart becomes constant and parameter k shows slightly increasing pattern for increasing shifts in the process. The power of the design is high for large shifts and type I error probability is low. If we need still better statistical properties, one may switch to economic-statistical design as proposed by Saniga (1989) or Celano (2011).

If the process quality characteristics have non-normal distribution, the optimum design parameters can be obtained in similar way. The out of control ARL (ARL_1) can be calculated based on an *out-of-control* probabilities $p = P(x_{ij} > \mu_0)$ for various shifts in the process as explained previously in section 2. These ARL_1 values will be used in further procedure of obtaining economic design parameters of EWMA control chart. Since, all further calculations uses ARL values based on *out-of-control* probabilities $p = P(x_{ij} > \mu_0)$ which are independent of process distribution, we will get similar performance as above for any process distribution.

To illustrate the sensitivity of the design for the errors in estimation of time and cost parameters, we have adopted the procedure by Kooli & Limam (2015). We have defined the lowest and highest levels of set S = {δ, θ, g, a, b, D, C, V, W}, which is the set of time and cost parameters along with shift parameter (δ) and exponential life time parameter (θ) of the process. These lowest and highest values of set S are given in Table 4. We have formed sixteen different

Table 3. Optimum design of EWMA control chart for different process shifts.

Shift	n	h	λ	k	α	power	loss
0.5	12	0.6	0.2	1.7	0.0399	0.309	19.060
0.8	12	0.8	0.2	1.7	0.0399	0.493	15.839
1.0	12	0.9	0.2	1.7	0.0399	0.601	14.600
1.2	12	1	0.2	1.7	0.0399	0.702	13.669
1.5	15	1.4	0.2	1.7	0.0414	0.931	12.527
1.7	15	1.4	0.2	1.7	0.0414	0.974	12.259
2.0	15	1.2	0.2	2.0	0.0220	0.957	11.729
2.2	15	1.3	0.2	2.0	0.0220	0.982	11.591
2.5	15	1.3	0.2	2.0	0.0220	0.996	11.515
3.0	15	1.2	0.2	2.3	0.0111	0.980	11.225

Table 4. Input levels of process and cost parameters.

Parameter	Level	
	Low	High
δ	1	2
θ	0.02	0.05
g	0	0.1
a	0.5	1.5
b	0.025	0.175
D	0.25	1.75
C	25	175
V	20	80
W	10	40

combinations of lowest and highest levels of parameters of set S. These combinations are presented in Table 5. First case in this table represents each parameter at lowest level and last case represents combination of all parameters at highest level. To carry out economic sensitivity of EWMA chart, optimal design parameters and economic as well as statistical performance of these sixteen combinations are studied under EWMA scheme. The optimum design of these sixteen cases is given in Table 6 from which one can choose the combination of particular statistical/economic interest.

The following Table 7 shows directional effects of different input parameters on different design parameters and loss cost from the design. The blank spaces in Table 7 show the mixed or robust behavior of particular design parameter or loss cost with respect to the change in corresponding time or cost parameter.

Table 5. Combinations of process and cost parameter.

Case	δ	θ	g	a	b	D	C	V	W
1	1	0.02	0	0.5	0.025	0.25	25	20	10
2	2	0.02	0.1	0.5	0.175	1.75	25	20	10
3	1	0.02	0.1	1.5	0.025	1.75	175	20	10
4	2	0.05	0	1.5	0.175	0.25	175	20	10
5	1	0.05	0	1.5	0.175	1.75	25	80	10
6	2	0.02	0.1	1.5	0.025	0.25	25	80	10
7	1	0.05	0.1	0.5	0.175	0.25	175	80	10
8	2	0.02	0	0.5	0.025	1.75	175	80	10
9	1	0.05	0.1	1.5	0.175	0.25	25	20	40
10	2	0.02	0	1.5	0.025	1.75	25	20	40
11	1	0.05	0	0.5	0.175	1.75	175	20	40
12	2	0.02	0.1	0.5	0.025	0.25	175	20	40
13	1	0.02	0.1	0.5	0.025	1.75	25	80	40
14	2	0.05	0	0.5	0.175	0.25	25	80	40
15	1	0.05	0	1.5	0.025	0.25	175	80	40
16	2	0.05	0.1	1.5	0.175	1.75	175	80	40

Table 6. Sensitivity of the EWMA control chart design for changes in input parameters.

case	Optimum design parameters				Economic / statistical performance		
	n	h	λ	k	α	power	Loss
1	17	2.3	0.2	1.6	0.0507	0.776	1.8536
2	5	3.5	0.2	1.3	0.0920	0.901	2.9011
3	5	0.9	0.2	1.3	0.0920	0.542	15.0510
4	11	1.0	0.2	1.6	0.0502	0.976	11.0130
5	24	3.8	0.2	1.7	0.0411	0.855	6.7307
6	9	1.7	0.2	2.8	0.0030	0.485	2.9643
7	3	0.2	0.2	2.4	0.0076	0.186	20.4200
8	11	0.4	0.2	3.2	0.0009	0.479	10.1390
9	9	3.1	0.2	1.0	0.1700	0.851	5.4292
10	16	3.0	0.2	2.1	0.0181	0.952	2.3941
11	16	1.1	0.2	1.2	0.1115	0.911	23.6710
12	7	0.9	0.2	1.5	0.0619	0.870	7.3115
13	11	0.9	0.2	2.9	0.0020	0.328	3.4122
14	6	1.0	0.2	2.6	0.0049	0.445	4.2666
15	29	0.7	0.2	2.1	0.0181	0.764	12.1870
16	5	0.5	0.2	2.4	0.0086	0.449	29.589

Table 7. Directional effects on design parameters and loss cost due to change in input parameter.

Parameter	δ	θ	g	a	b	D	C	V	W
n	+ve	−ve	−ve	−ve	−ve			+ve	
h		−ve	+ve	+ve	+ve	+ve	−ve	+ve	
λ									
k	+ve	−ve	−ve	−ve	−ve	−ve		+ve	
loss	−ve	+ve				+ve	+ve		

We observe the following from Table 6 and Table 7, as far as the sensitivity of the design is concerned.

i. There is no significant effect on design parameters n and k of the EWMA design due to change in time and cost parameters C, D and W. The design parameter λ becomes robust for changes in all the parameters of set S.

ii. The loss cost is more sensitive to change in parameters δ, θ, C and D, but have negligible effect due to change in other parameters. All the design parameters as well as loss cost remain unchanged due to change in parameter W of set S. As power of the design increases, loss cost also increases.

iii. The sampling interval length increases with increasing values of input parameters a and b. It shows opposite pattern for change in the values of parameters C and D.

iv. The loss cost is large when all the input parameters are at higher level and opposite result is observed in reverse case. It is the general observation that, whenever Type I error probability (α) is low, power of the design is low.

v. Overall the design is not much cost sensitive to change in input parameters except for the values of parameters δ, θ, C and D.

6. Conclusions

In the present study we have developed an economic design of EWMA control chart based on sign statistic to control the location of the process. An expression for loss cost per unit time is obtained and is minimized with respect to the design parameters of EWMA control chart. It is observed that, the chart is more economic for large shifts in the process quality characteristic. For small shifts, we may adopt some specific value of λ and k which will provide desirable power at slight increase in loss cost and the design become economic statistical. Under an economic condition, as shift in the process increases sample size required to detect the shift decreases.

The economic design is sensitive to change in shift (δ), parameter of *in-control* life time distribution (θ), the values of penalty cost due to non-conformities produced (C) and time required for search and repair of an assignable cause (D). The moderate sensitivity is observed for the changes in other cost and time parameters. The economic design is fairly robust for the cost of repair (W) of the process and the design parameter λ of EWMA control chart. Power of the chart seems to be good under economic optimal condition. This economic design can be applied as like nonparametric procedure for the production processes whose quality distribution is unknown.

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Economic design of variable sampling interval sign control chart

S. H. Patil[a] and D. T. Shirke[b]

[a]Department of Statistics, Doodhsakhar Mahavidyalaya, Bidri, India; [b]Department of Statistics, Shivaji University, Kolhapur, India

ABSTRACT
In this article, we have proposed an economic design of variable sampling interval (VSI) control chart based on sign statistic. An expression for expected loss per unit time is obtained and minimized with respect to design parameters. The performance of this economic design is compared with economic design of fixed sampling interval (FSI) sign chart to control location of process characteristic. Though the power of VSI sign chart is slightly lower than that of FSI sign chart design, it appears to be dominant over FSI sign chart design for moderate shifts. The effect of change in input parameters on loss cost is observed for different out of control probabilities of quality characteristic under VSI sign chart scheme. Change in shift parameter, penalty cost parameter, and time to detect the shift have considerable effect on loss cost from the design. The design is fairly robust to the changes in other parameters.

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Economic design; nonparametric control chart; sign control chart; variable sampling interval design

1. Introduction

The control chart procedure was first introduced by Shewhart in 1931, as an online tool to detect whether the process is under statistical control or not. Since then it is mostly used in many production processes to detect *out-of-control* situation in the process so that, the causes responsible for it can be detected and removed to overcome defective product. It is a common practice that, the process is assumed to be start in *in-control* state. After some random time period, it shifts to an *out-of-control* situation due to occurrence of an assignable cause. In due course, the control chart of quality characteristic is plotted, which at some instance of sampling signals the *out-of-control* situation. Afterwards, the cause is detected, removed and the process is brought back again in control. The time from starting of the process up to this stage of bringing process back in control, when an assignable cause occurs is termed to be one production cycle. After this a fresh cycle begins.

In economic design of control chart, an expression for expected loss or gain (as a case may be) per unit time or per units produced is obtained and is optimized (minimized or maximized) with respect to the design parameters of the control chart under consideration. Duncan [1] has first proposed an economic design of $\overline{X}$ control chart using numerical optimization of loss cost function during a production cycle. Since then, many other researchers have worked on economic designs of different types of control charts, survey of which is conducted by Montgomery.[2] The survey consists of the summary of the work done by near about 51 researchers including Aroian and Levene,[3] Bather,[4] Chou,[5, 6] Chou and Wetherill,[7] Duncan,[8] Gibra,[9, 10] Goel and Wu,[11] Taylor,[12] Saniga[13], and many more. The survey consists of the economic design of variables as well as attributes control charts. After 1980s Lorenzen and Vance,[14] Rahim,[15] Koo and Case [16] have proposed economic designs of control charts using different approaches.

The most common thing in above referred work is that, the quality characteristic of the data is assumed to be normally distributed providing the distribution of sample mean becomes normal. During 1990s and thereafter, the concept of adaption of design parameters came forward at the same time concentration is made on non-normality nature of the quality characteristic of the data. Reynolds et al. [17] have worked on adaptive control charts and commented that, these control charts work better as compared to traditional control charts. Adaptive control charts provide better statistical performance as compared to traditional charts in terms of power, average time to signal (ATS) etc. of the chart. Considering these facts, Reynolds et al. [17], Bai, and Lee [18] have developed economic design of VSI $\overline{X}$ control charts. They have concluded that VSI feature of $\overline{X}$ control chart substantially reduces ATS of the chart for small to moderate shifts in the process. Mahadik and Shirke [19] have developed economic design for modified variable sample size and sampling interval $\overline{X}$ control chart. They have illustrated an economic application of

CONTACT S. H. Patil psubhash_2007@rediffmail.com

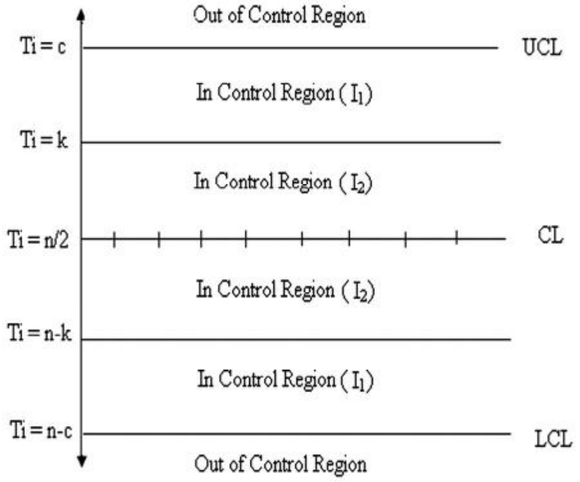

Figure 1. The VSI sign control chart.

adaptive control chart considering data from Lorenzen and Vance [14]. Rahim [20] has developed an economic design under non-normality and measurement errors. Chou et al. [21] have developed economic statistical design for averages chart under non-normality. Chen [22] has developed economic design of $\overline{X}$ chart under non-normality using variable sampling interval (VSI). Patil and Shirke [23] have developed economic design of moving average control chart using VSI policy for the data with non-normal quality characteristics. Some charts for variable parameter are also developed. In all these and other charts, if the quality characteristic is non-normal, is transferred to normal using transformations like Burr [24] or Johnson [25] etc. Due to such data transformations, there may be possibility of lack of information; hence the other alternative remain is to use non-parametric control charts proposed by Amin et al. [26], Amin and Widmaier,[27] Bakir [28] etc., which are based on distribution free methods.

In the present study, an attempt is made to obtain an economic design of control chart based on non-parametric sign statistic using VSI proposed by Amin and Widmaier.[27] Theoretical expression for expected loss cost during a production cycle is derived and is optimized using numerical example. The organization of paper consists of eight sections. Section 2, followed by present section describes construction of sign and VSI sign control charts. The process design and notations used are given in Section 3. The derivations for expected cycle length and expected loss cost during a production cycle are derived in Sections 4 and 5 respectively. An example and the sensitivity of the design are carried out respectively in Sections 6 and 7. Final conclusions are given in section 8 followed by the references.

2. The sign and vsi sign control charts

Amin et al. [26] have explained in detail the design of sign control chart and its advantages in statistical process control. When the distribution of quality characteristic is asymmetric, then the sign control chart is actually the chart used to measure the change in the value of process median. Hence, mostly the process target value μ_0 of the sign control chart is taken as the value of in-control median of the process quality characteristic. The samples are drawn at regular intervals from the process. Let i denotes the time instant at which sample is drawn and j denotes observation number in that sample so that, x_{ij} denotes jth observation in the ith sample from the process with target median μ_0 ($i = 1, 2, \ldots \ldots$; $j = 1, 2, \ldots \ldots, n$).

Define,

$$\mathrm{SN}_i = \sum_{j=1}^{n} \mathrm{sign}\left(x_{ij} - \mu_0\right), \quad i = 1, 2, \ldots \ldots \ldots$$

where,

$$\mathrm{sign}(x) = \begin{cases} -1 & ; \text{if } x < 0 \\ 0 & ; \text{if } x = 0 \\ +1 & ; \text{if } x > 0 \end{cases}$$

The random variable $T_i = (\mathrm{SN}_i + n)/2$, then gives number of positive signs in a sample of size n and have binomial distribution with parameters (n, p), where $p = P(x_{ij} > \mu_0)$. It is easy to give the decision based on statistic T_i. As long as the process median is at μ_0, $p = 1/2$ and $E(\mathrm{SN}_i) = 0$ providing $E(T_i) = n/2$. The chart signals a shift in the process if $T_i > c$ or $T_i < n - c$, where c is a positive constant fairly known to be parameter of sign control chart. Average run length of the chart is given by $L(\mu) = 1/P(T_i > c$ or $T_i < n - c)$ for two-sided chart and for positive one-sided chart it is taken to be $L^{+}(\mu) = 1/P(T_i > c)$.

Amin and Widmaier [27] have explained the construction of two-sided VSI sign control chart by considering two sampling intervals. They used ATS to measure the efficiency of the control chart. Considering the fact that the process average shifts in between the samples, for practical use, they have provided an expression for adjusted average time to signal (AATS), which is the time between actual occurrence of shift in the process and the event of detection of the shift.

In VSI sign chart, the statistic used is, $T_i = (\mathrm{SN}_i + n)/2$, which gives number of (+) signs in a sample and have only discrete values. The large number of (+) signs in the chart supports to upward shift whereas small number of (+) signs in the chart are suppose to support for downward shift in the process target value. Whenever the number of (+) signs are not too large nor too small, suggest waiting for more time for the next sample and whenever the number of (+) signs occurs in the warning region (that is, near to *out-of-control* limits), suggests to take a sample in quicker time. The control chart depends on number of (+) signs and the *in-control* region (n–c, c) is divided in two sub regions I_1 and I_2, given by,

$I_1 = [(n-c), (n-c+1), \ldots\ldots (n-k-1)] \, U$
$[(k+1), (k+2), \ldots\ldots c]$
$I_2 = [(n-k), (n-k+1), \ldots\ldots\ldots k]$

where n, k and c denotes number of (+) signs in sample such that $0 < k < c < n$. Figure 1 represents pictorial presentation of this VSI sign control chart.

If the control statistic T_i (that is, number of (+) signs) falls in region I_1, wait for period h_1 units of time for the next sample and if it falls in region I_2, wait for period h_2 units of time for the next sample ($h_1 < h_2$), otherwise the process is shifted to the *out-of-control* state. Like most of the VSI design procedures, it is preferred to use the shortest sampling interval length (h_1) at the beginning of the process, as it provides more protection against the problems arising at start up [refer Bai and Lee [18], Yu and Wu [29]]. The constants, (n, h_1, h_2, k, c) can be referred to be as the parameters of this VSI sign control chart design.

3. Process design

Consider the production process monitored by drawing a sample of size n at an interval of every h hour ($h = h_1$ or h_2). The process is monitored by a VSI sign control chart design with single assignable cause and the process target value as the median of the quality characteristic distribution of the process denoted by μ_0. Whenever an assignable cause occurs, the target value shifts from μ_0 to $\mu_0 + \delta\sigma$, where σ is the process standard deviation and δ is the amount of shift in the process in σ units, otherwise it remains at μ_0 [Patil and Shirke [23]]. The process is assumed to be start in *in-control* state and will shift to an *out-of-control* state in near future. The transaction between *in-control* and *out-of-control* states is assumed to be instantaneous. The notations used are as follows.

3.1. Notations

n: sample size.
h_1, h_2: sampling intervals in hours of the two-sided VSI sign control chart.
k, c: the control limit parameters of the two sided VSI sign control chart.
λ: parameter of an exponential life time distribution for the *in-control* state.
α: probability of false alarm.
D: expected search and repair time of an assignable cause during the true alarm.
ICT: expected *in-control* time of the process.
$E(T)$: expected time period of a production cycle.
C: expected penalty cost per unit time due to running the process in *out-of-control* state.
AATS: adjusted average time to signal for the two-sided VSI sign control chart.
$E(A)$: expected income from the process during a production cycle.
$E(I)$: expected income per unit time from the process during a production cycle.
$E(L)$: expected loss per unit time from a process during a production cycle.
N: number of samples drawn during a production cycle.
g: time to sample, inspect, and conclude one unit in the sample.
a, b: fixed and variable costs of sampling, respectively.
V: loss cost due to a single false alarm.
W: loss cost for search and repair of an assignable cause.

4. Expected cycle length

Expected cycle length consists of an *in-control* period, *out-of-control* period, time for sampling and testing and the time for search and repair of an assignable cause. Assuming that, an assignable cause occurs according to a Poisson process of an intensity of λ occurrences per unit time, the expected *in-control* time (ICT) becomes $1/\lambda$.

Therefore,

$$\mathrm{ICT} = \frac{1}{\lambda} \tag{1}$$

As explained above in Section 2, let us assume that, the process is monitored by two sampling interval sign control chart. Since T_i has binomial distribution with parameters (n, p) and when the process is under control $p = 0.5$, according to Amin and Widmaier [27] the probabilities that sample statistic falls in these two sub regions (I_1 and I_2), when the process is under control are,

$$p_{0j} = P(T_i \in I_j / p = 0.5), \quad ; \quad j = 1, 2$$

$$p_{0j} = \sum_{t \in I_j} \binom{n}{t} 0.5^n \quad ; \quad j = 1, 2. \tag{2}$$

This implies,

$$p_{01} = \sum_{t=n-c}^{n-k-1} \binom{n}{t} 0.5^n + \sum_{t=k+1}^{c} \binom{n}{t} 0.5^n,$$

$$= 2 \sum_{t=k+1}^{c} \binom{n}{t} 0.5^n.$$

$$p_{02} = \sum_{t=n-k}^{k} \binom{n}{t} 0.5^n.$$

Similarly, the probabilities that, the sample statistic falls in two regions (I_1 and I_2), when the process is *out-of-control* are,

$$p_{1j} = P(T_i \in I_j / p \neq 0.5), \quad ; j = 1, 2$$

$$p_{1j} = \sum_{t \in I_j} \binom{n}{t} p^t (1-p)^{n-t} \quad ; \quad j = 1, 2. \tag{3}$$

Hence, the probability of false alarm (α) and the power ($1-\beta$) of the chart is given by,

$$\begin{aligned}\alpha &= P\,(\text{chart signal/ shift does not occurs})\\ &= P(T_i \in out-of-control\ region/p = 0.5)\\ &= \sum_{t=0}^{n-c-1}\binom{n}{t}0.5^n + \sum_{t=c+1}^{n}\binom{n}{t}0.5^n. \end{aligned} \tag{4}$$

and,

$1-\beta = P(\text{chart signals/ shift occurs})$

$1-\beta = P(T_i \in \textit{out-of-control}\ \text{region}/p \neq 0.5)$,

$$1-\beta = \sum_{t=0}^{n-c-1}\binom{n}{t}p^t(1-p)^{n-t} + \sum_{t=c+1}^{n}\binom{n}{t}p^t(1-p)^{n-t}. \tag{5}$$

According to Amin and Widmaier [27] the AATS is given by,

$$\text{AATS} = \frac{\sum_{j=1}^{2} h_j^2 p_{0j}}{2\sum_{j=1}^{2} h_j p_{0j}} + \frac{1}{1-\beta}\sum_{j=1}^{2} h_j p_{1j}. \tag{6}$$

Since, g is the time for testing and sampling per sample unit and D is time for search and repair of an assignable cause, the expected time period of production cycle $E(T)$ is,

$$E(T) = 1/\lambda + \text{AATS} + gn + D. \tag{7}$$

5. Expected loss cost function during the cycle

The loss cost occurred during the cycle consists of the loss cost due to nonconformities produced during a production cycle, loss cost due to search during the false alarm, cost for search and repair of an assignable cause and the cost for sampling and testing.

Let V_0 and V_1 be the net income per unit time from the process during *in-control* and *out-of-control* state, so that $C = V_0 - V_1$ is the penalty loss cost due to running the process in *out-of-control* state.

The expected income from the process is now given by,

$$E(A) = \frac{V_0}{\lambda} + V_1\left[\text{AATS} + gn + D\right],$$

This can be written as,

$$E(A) = \frac{V_0}{\lambda} + V_0\left[\text{AATS} + gn + D\right] - C\left[AATS + gn + D\right].$$

This gives expected income per unit time during the production cycle as,

$$E(I) = \frac{E(A)}{E(T)},$$

$$= V_0 - \frac{C\left[\text{AATS} + gn + D\right]}{E(T)}.$$

Therefore, the loss cost per unit time (L_1) due to non conformities produced during the cycle is given by,

$$L_1 = V_0 - E(I) = \frac{C\left[\text{AATS} + gn + D\right]}{E(T)}. \tag{8}$$

Let, S_0 and S_1 denotes expected number of samples taken during *in-control* period and *out-of-control* period, respectively, then,

$$S_0 = \frac{1/\lambda}{E(h)}.$$

where $E(h)$ is the expected length of sampling interval and according to Bai and Lee [18],

$$S_1 = \frac{1}{1-\beta}.$$

Since the probability of false alarm is α, the loss cost per unit time (L_2) due to false alarm and repair of the process during true alarm is,

$$L_2 = \frac{V\alpha S_0 + W}{E(T)}. \tag{9}$$

Now, the expected numbers of samples during the cycle are,

$$E(N) = S_0 + S_1.$$

Therefore, the cost for sampling and testing per unit time (L_3) is,

$$L_3 = \frac{(a+bn)E(N)}{E(T)}. \tag{10}$$

Hence, the expected total loss cost per unit time during the cycle is,

$$E(L) = L_1 + L_2 + L_3. \tag{11}$$

Equation (11) represents the equation for the loss cost per unit time during a production cycle. This loss cost should be minimized with respect to the design parameters of a VSI sign control chart. Several numerical approximation procedures such as partial derivative method, iterative procedure, pattern search method etc. are available in the literature for optimization of objective functions. Instead of using numerical approximation, for easy optimization, here we have developed a simple MATLAB code to obtain the values of optimum design parameters. A code in MATLAB is written by considering loss cost function in (11) as an objective function and different parameters as constraints on this objective function. This code is processed by considering different values of input parameters in valid interval which in turn gives rise to an optimal solution to the objective function.

In MATLAB, there is optimization function "fmincon" which gives optimum solution for the objective function under given constraints. For the use of this function, we have to provide initial values of parameters. Mostly, it gives global optimum but sometimes may give different results. Hence, we have developed a MATLAB code based on pattern search method which provides unique optimum solution within few minutes. In this method, possible combinations of parameter are obtained and the value of objective function is evaluated for each combination. From these values optimum can be obtained. In the proposed example in next section, we have written a code in MATLAB to minimize $E(L)$ under the constraints $0 < n < 50$, $0.01 < h_1 < h_2 < 10$ and $1 < k < c < n$ which ultimately gives optimum design for VSI sign control chart.

6. An example

To illustrate the design, consider an example of non-returnable glass bottles production process given by Montgomery [30]. The time and cost parameters used are $g = 0.0167$ hour, $D = 1$ hour, $C = \$100$, $a = \$1$, $b = \$0.1$, $V = \$50$ and $W = \$25$. The *in-control* run length parameter is $\lambda = 0.05$ and shift parameter is $\delta = 2$. We will apply the economic procedure using VSI sign chart to the same input data. When the process target value is the median of the process, for *in-control* state of the process, the probability $p = Pr(x_{ij} > \mu_0)$ will be 0.5 (Say p_0). When the process shifts to an *out-of-control* state, the value of p shifts from p_0 to p_1 ($p_1 > 0.5$). It should be noted that, as a shift in the process increases, the value of p_1 goes on increasing. Here, assuming different values for p_1, in the range 0.53–0.99, we have obtained optimum design parameters (n, h_1, h_2, k, c) and the minimum loss cost for the VSI sign control chart to control the median. The results obtained are given in Table 1.

Table 1 reveals that, as shift in the process location increases, power of the VSI sign control chart to detect the shift increases. As a result loss cost from the process decreases with increasing shift in the process. The sample size to detect the shift is slightly large for moderate shift and parameters k and c changes accordingly. Sampling interval h_1 decreases with increasing shift and h_2 varies around 1 h. Type I error probability decreases and power of the chart increases with increasing shift provides, the VSI sign chart works better for large shifts.

It is observed that, the design parameters n, h_1, k, and c are robust to the change in *in-control* parameter λ. As a result the type I error and power of the design remain unchanged. On the other hand, considerable effect is observed on the parameter h_2 and the minimum loss cost due to change in parameter λ. The sampling interval (h_2) decreases and the loss cost increases with increasing values of λ. As a sample, observe following Table 2, for different values of λ and $p_1 = 0.97$.

For easy comparison between optimum VSI and fixed sampling interval (FSI) designs, both the optimum sign chart designs be presented in a single table. We have considered one-sided sign control chart design for both the processes so that, the comparison will be made on the same platform. Table 3 below gives summary of the comparison of optimum VSI Sign chart design with FSI Sign chart design for different p_1 values. This table is also useful for the comparison of one-sided and two-sided optimum VSI sign control chart design.

Table 1. Optimum design of VSI sign control chart for different shifts in location.

P_1	n	h_1	h_2	k	c	α	Power	Loss
0.53	9	0.9	0.9	5	6	0.1797	0.1868	34.5604
0.55	9	0.9	1.0	5	6	0.1797	0.1993	33.7247
0.58	10	0.2	0.8	6	7	0.1094	0.1543	31.8371
0.60	10	0.1	0.9	6	7	0.1094	0.1796	29.9610
0.65	10	0.1	1.1	6	7	0.1094	0.2664	25.4516
0.70	9	0.1	1.0	5	7	0.0391	0.1964	21.2559
0.75	10	0.1	0.9	6	8	0.0215	0.2441	17.4479
0.80	10	0.1	1.1	6	8	0.0215	0.3758	14.8758
0.85	10	0.1	1.3	6	8	0.0215	0.5443	13.5067
0.90	9	0.1	0.9	6	8	0.0039	0.3874	12.4069
0.95	10	0.1	1.0	7	9	0.0020	0.5987	11.4724
0.97	9	0.1	1.0	7	8	0.0039	0.7602	11.1591
0.99	9	0.1	1.0	7	8	0.0039	0.9135	10.9264

Table 2. Effect of change in λ on the economic design of VSI sign control chart.

λ	n	$h1$	$h2$	k	c	α	Power	Loss
0.02	9	0.1	1.5	7	8	0.0039	0.7602	5.7367
0.03	9	0.1	1.2	7	8	0.0039	0.7602	7.6797
0.04	9	0.1	1.1	7	8	0.0039	0.7602	9.4748
0.05	9	0.1	1.0	7	8	0.0039	0.7602	11.1591
0.06	9	0.1	0.9	7	8	0.0039	0.7602	12.752
0.07	9	0.1	0.8	7	8	0.0039	0.7602	14.2763
0.08	9	0.1	0.8	7	8	0.0039	0.7602	15.7400

From Table 3, we can conclude that,

(1) The sample size required to detect the shift in the process is appears to be small for VSI sign chart design as compared to FSI sign chart design.
(2) The type I error (α) and power of VSI sign chart are small as compared to FSI sign chart.
(3) The control limit parameter (c) is large for VSI sign control chart design as compared to FSI design.
(4) The saving due to VSI sign chart design is large for moderate shifts in the process as compared to small and large shifts in the process. Moreover, FSI procedure appears to be more economic as compared to VSI for small shifts in the process.

It is observed from Tables 1 and 3 that, the one-sided VSI sign chart procedure is more economic as compared to two-sided VSI design. The optimum sample size for two-sided chart is small as against one sided chart. The power of both these chart is low as compared to FSI one-sided chart. Though the one-sided VSI sign chart appears to be more economic, for sensitivity study we are going to use only two-sided VSI control chart design as our process design is based on two-sided chart procedure. Also, these two charts perform in similar way except only for change in values of loss cost.

Table 3. Comparison between optimum FSI and VSI Sign control chart design.

	VSI Design								FSI Design						% saving in
p_1	n	h_1	h_2	k	c	a	Power	Loss	n	h	c	a	Power	Loss	loss
0.55	17	0.01	1.4	9	10	0.1662	0.2902	29.189	15	2.7	2	0.5	0.6535	26.036	−12.109
0.58	21	0.01	1.3	11	13	0.0946	0.283	24.867	27	2.4	3	0.3506	0.6771	24.398	−1.9219
0.60	21	0.01	1.5	11	13	0.0946	0.3495	22.748	32	1.8	6	0.1885	0.6039	22.9851	1.0311
0.65	27	0.01	1.5	14	18	0.0261	0.3577	17.881	30	1.5	8	0.1002	0.6548	19.7792	9.59644
0.70	21	0.01	1.4	11	15	0.0133	0.3627	15.51	27	1.4	9	0.061	0.7276	17.4301	11.0166
0.75	17	0.01	1.3	9	13	0.0064	0.353	14.023	24	1.2	10	0.032	0.7662	15.7121	10.7529
0.80	14	0.01	1.2	8	11	0.0065	0.4481	12.93	20	1.1	10	0.0207	0.8042	14.416	10.3066
0.85	12	0.01	1.1	7	10	0.0032	0.4435	12.149	15	1.1	9	0.0176	0.8227	13.379	9.19202
0.90	8	0.01	0.9	5	7	0.0039	0.4305	11.46	13	1	9	0.0112	0.8661	12.5184	8.45396
0.95	8	0.01	1	5	7	0.0039	0.6634	11.038	10	1	8	0.0107	0.9139	11.7411	5.98837

Table 4. Effect due to change in time and cost parameters on minimum loss cost.

Parameter	Value	n	h_1	h_2	k	c	a	Power	Loss	% change in loss
C	25	9	0.1	2.0	7	8	0.003906	0.7602	4.7427	−57.4993
	50	9	0.1	1.4	7	8	0.003906	0.7602	7.0677	−36.6642
	75	9	0.1	1.1	7	8	0.003906	0.7602	9.1677	−17.8455
	125	9	0.1	0.8	7	8	0.003906	0.7602	13.0766	17.18329
	150	9	0.1	0.8	7	8	0.003906	0.7602	14.935	33.83696
	175	9	0.1	0.7	7	8	0.003906	0.7602	16.7574	50.16802
a	0.25	9	0.1	0.8	7	8	0.003906	0.7602	10.2735	−7.93612
	0.5	9	0.1	0.8	7	8	0.003906	0.7602	10.5884	−5.11421
	0.75	9	0.1	0.9	7	8	0.003906	0.7602	10.8799	−2.50199
	1.25	9	0.1	1.0	7	8	0.003906	0.7602	11.413	2.275273
	1.5	9	0.1	1.1	7	8	0.003906	0.7602	11.6607	4.494986
	1.75	9	0.1	1.1	7	8	0.003906	0.7602	11.8924	6.571318
b	0.025	9	0.1	0.8	7	8	0.003906	0.7602	10.3679	−7.09018
	0.05	9	0.1	0.8	7	8	0.003906	0.7602	10.6513	−4.55055
	0.075	9	0.1	0.9	7	8	0.003906	0.7602	10.908	−2.25018
	0.125	9	0.1	1.0	7	8	0.003906	0.7602	11.3876	2.047656
	0.15	8	0.1	1.1	6	7	0.007813	0.7837	11.5855	3.821097
	0.175	8	0.1	1.1	6	7	0.007813	0.7837	11.7761	5.52912
V	12.5	7	0.1	1.0	5	6	0.015625	0.808	10.8665	−2.62208
	25	8	0.1	1.0	6	7	0.007813	0.7837	10.9906	−1.50998
	37.5	8	0.1	1.0	6	7	0.007813	0.7837	11.0868	−0.6479
	62.5	9	0.1	1.0	7	8	0.003906	0.7602	11.2057	0.417596
	75	9	0.1	1.0	7	8	0.003906	0.7602	11.2524	0.836089
	87.5	9	0.1	1.0	7	8	0.003906	0.7602	11.299	1.253685
W	6.25	9	0.1	0.9	7	8	0.003906	0.7602	10.2952	−7.74166
	12.5	9	0.1	1.0	7	8	0.003906	0.7602	10.5834	−5.15902
	18.75	9	0.1	1.0	7	8	0.003906	0.7602	10.8713	−2.57906
	31.25	9	0.1	1.0	7	8	0.003906	0.7602	11.4469	2.579061
	37.5	9	0.1	1.0	7	8	0.003906	0.7602	11.7348	5.159018
	43.75	9	0.1	1.0	7	8	0.003906	0.7602	12.0226	7.738079
g	0.0042	9	0.1	1.0	7	8	0.003906	0.7602	10.6964	−4.14639
	0.0084	9	0.1	1.0	7	8	0.003906	0.7602	10.8524	−2.74843
	0.0125	9	0.1	1.0	7	8	0.003906	0.7602	11.0042	−1.3881
	0.0209	9	0.1	1.0	7	8	0.003906	0.7602	11.3135	1.383624
	0.0251	8	0.1	1.0	6	7	0.007813	0.7837	11.4575	2.674051
	0.0292	8	0.1	1.0	6	7	0.007813	0.7837	11.5908	3.868592
D	0.25	9	0.1	0.9	7	8	0.003906	0.7602	7.9744	−28.539
	0.5	9	0.1	0.9	7	8	0.003906	0.7602	9.0617	−18.7954
	0.75	9	0.1	0.9	7	8	0.003906	0.7602	10.1235	−9.28032
	1.25	9	0.1	1.0	7	8	0.003906	0.7602	12.1703	9.061663
	1.5	9	0.1	1.0	7	8	0.003906	0.7602	13.1588	17.9199
	1.75	9	0.1	1.0	7	8	0.003906	0.7602	14.1253	26.581

7. Sensitivity of the design

To illustrate sensitivity of the design for the errors in estimation of time and cost parameters, we have changed time and cost parameters by ±25%, ±50%, ±75 and observed the change in minimum loss cost and design parameters of the VSI sign control chart. For the purpose of illustration, we have considered only one value of p_1 (0.97) having optimal design parameters $n = 9$, $h_1 = 0.1$, $h_2 = 1.0$, $k = 7$, $c = 8$ with optimal loss cost \$11.1591 corresponding to inputs $C = \$100$, $a = \$1$, $b = \$0.1$, $V = \$50$, $W = \$25$, $g = 0.0167$ and $D = 1$. We have changed value of one of these input values keeping other constant and observed the effect on loss cost and design parameters of the chart. The outputs are presented in Table 4. In the table the last column (% change in loss) gives percentage increase or decrease in the optimal loss cost due to change in the particular input parameter with respect to optimal loss \$11.1591.

We observe from Table 4 that, there is no effect on design parameters n, k, and c due to change in input parameters *C*, *a*, *W*, and *D*. Parameter h_1 is insensitive but parameter h_2 shows considerable effect due to change in time and cost parameters. There is significant effect on loss cost due to change in input parameters *C* and *D*, but incompetent effect is observed due to change in parameters *a*, *b*, *V*, *W*, and *g*. In all the design is observed to be sensitive for the change in the values of penalty loss cost due to non-conformities produced (*C*) and time for search and repair of an assignable cause during true alarm (*D*).

8. Conclusions

In the present study, we have developed an economic design of VSI sign control chart and compared it with economic design of usual FSI sign control chart. We observed that, the VSI sign control chart is more economic to FSI sign chart for moderate shifts in the process. The power of VSI sign chart is less than that of FSI sign chart. The FSI sign chart is more economic for small shifts and is slightly expensive as compared to VSI sign chart for large shifts in the process. Moreover, one-sided VSI sign control chart is more economic than two-sided VSI sign chart. As like FSI sign control chart, the power of VSI sign control chart also increases for large shifts in the process.

The change in *in-control* parameter (λ), highly affects on the loss cost from the process. Smaller values of λ, reduce the values of loss cost from the process. The type I error probability and power does not have much effect of change in parameter λ. The VSI sign chart design is sensitive to errors in the estimation of penalty cost due to non-conformities produced (*C*) and time for search and repair of an assignable cause during true alarm (*D*). The moderate sensitivity is observed in case of parameters *a*, *b*, *W*, and *g*. As far as, the loss is concerned, the design is nearly robust to the errors in estimation of parameter *V*. The theoretical base of the study and results from example supports application of the procedure to any real life data, probably having unknown, and/or heavy tailed distribution.

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Disclosure statement

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Notes on contributors

S. H. Patil is an assistant professor in Statistics at Doodhsakhar Mahavidyalaya, Bidri, Kolhapur, Maharashtra, India. He received his master's degree in Statistics from the Shivaji University, Kolhapur. He is pursuing his PhD at the Department of Statistics, Shivaji University, Kolhapur. His research interests include statistical quality control and optimization.

D. T. Shirke is a professor of Statistics, working in Shivaji University, Kolhapur, Maharashtra, India. He received his PhD in Statistics from the Shivaji University, Kolhapur. His research interests include statistical process control, statistical inference, and data mining. He has published several research papers in national and international journals.

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RESEARCH ARTICLE

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A nonparametric CUSUM chart for process dispersion

D.T. Shirke | M.S. Barale

Department of Statistics, Shivaji University, Kolhapur 416004, Maharashtra, India

Correspondence
M. S. Barale, Department of Statistics, Shivaji University, Kolhapur 416004, Maharashtra, India.
Email: baralemahesh12@gmail.com

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Abstract

In the present article, we propose a nonparametric cumulative sum control chart for process dispersion based on the sign statistic using in-control deciles. The chart can be viewed as modified control chart due to Amin et al,[6] which is based on in-control quartiles. An average run length performance of the proposed chart is studied using Markov chain approach. An effect of non-normality on cumulative sum S^2 chart is studied. The study reveals that the proposed cumulative sum control chart is a better alternative to parametric cumulative sum S^2 chart, when the process distribution is non-normal. We provide an illustration of the proposed cumulative sum control chart.

KEYWORDS

average run length, nonparametric chart, process control, sign test

1 | INTRODUCTION

Control charts are used to monitor process parameter, such as location, dispersion, and proportion of defectives. The widely used control charts are $\bar{X}$ chart for process location and R chart or S^2 chart for the process dispersion; these charts are also known as Shewhart's charts. The main drawback of these charts is that these charts are less efficient against small shifts. One can resolve this problem by using runs rules. There are some operational issues while implementing the runs rules charts. An alternative to detect the small shift is to use memory chart, as like cumulative sum (CUSUM) chart proposed by Page[1] or exponentially weighted moving average (EWMA) charts proposed by Roberts.[2] These charts consider the past as well as current information about the process, which makes charts very sensitive to small shifts in the process parameters. In the literature, various parametric CUSUM procedures are available for monitoring process location and dispersion, but very few nonparametric CUSUM procedures are available to monitor the process dispersion.

The traditionally used CUSUM S and CUSUM S^2 charts are based on the assumption of normality, but when the process distribution is not normal, the false alarm probability of the chart varies. Therefore, one of the robust alternatives to these charts is to use the nonparametric control charts. A control chart is said to be nonparametric, if its in-control average run length (ARL) does not depend on underlying process distribution. The performance of a control chart is usually measured using the ARL, which is defined as an average number of samples required to get an out-of-control signal.

Till date, there are several parametric as well as nonparametric Shewhart's control charts reported in the literature for process location and dispersion. Bakir[3] developed a distribution-free Shewhart quality control chart based on a signed-rank like statistic for process location. Chakraborti and Eryilmaz[4] have proposed a control chart based on a signed-rank statistic for process center. Khilare and Shirke[5] developed a nonparametric synthetic control chart using a sign statistic for process location. Amin et al[6] developed a nonparametric control chart based on sign statistic for the process center and variability. They have also developed CUSUM chart by using sign statistic for process center and reported that it can be extended for variability also. Rendtel[7] and Reynolds et al[8] described a CUSUM chart with variable sampling intervals for process mean. Yang and Cheng[9] have proposed a nonparametric CUSUM mean chart based on the sign statistic. Das[10] developed a nonparametric control chart for variability based on the squared rank statistic. Khilare and Shirke[11] have proposed a nonparametric synthetic control chart for the process variation. Shirke et al[12] have proposed a nonparametric control chart for process variability based on in-control deciles. Zhou et al[13] provided a nonparametric quality control chart based on Ansari-Bradly test statistic for variability. Chowdhury et al[14] constructed a nonparametric control chart for joint location and scale monitoring, which is based on the Lepage test. Guo and Wang[15] have proposed a variable sampling interval S^2 chart with known or unknown in-control variance. Zombade and Ghute[16] provided 4 nonparametric control charts for the process variation based on Sukhatme's 2 sample test and Mood's test. In the proposed work, we propose a CUSUM chart based on sign statistic defined by Shirke et al.[12] The sign statistic is defined using in-control deciles of the process distribution.

The remaining article is organized as follows. Section 2 describes the effect of non-normality on S^2 chart, a nonparametric CUSUM chart based on in-control deciles, and method for obtaining its ARL. Section 3 provides the performance study of control charts for various process distributions. Sections 4 and 5 provide the illustrative example and conclusions, respectively.

2 | A NONPARAMETRIC CUSUM CHART BASED ON IN-CONTROL DECILES

Suppose we are monitoring the process for detecting variation in the quality characteristic of interest say X. Let variance of X is σ^2 and when the process is in-control $\sigma^2 = \sigma_0^2$. We monitor the process by drawing a random sample of size n at fixed time epoch. Let X_{ij} be the j^{th} observation at time epoch i, where $i = 1, 2, \ldots$ and $j = 1, 2, \ldots, n$. In the literature, parametric CUSUM S^2 chart is used to monitor small changes in process dispersion, where S^2 be the sample variance. We have to detect a shift in process dispersion quickly. Let σ_1^2 be the process variance after change in the process dispersion.

The charting statistic for CUSUM S^2 chart are as follows:

$$\begin{aligned} C_i^+ &= max(0, S_i^2 - k + C_{i-1}^+) \\ C_i^- &= max(0, k - S_i^2 + C_{i-1}^-), \end{aligned} \tag{1}$$

where $k = [2ln(\sigma_0/\sigma_1)\sigma_0\sigma_1/(\sigma_0 - \sigma_1)]$ and $C_0^+ = C_0^- = 0$. The statistic C_i^+ and C_i^- are called as an upper and lower CUSUM's respectively and initial values of C_i^+ and C_i^- are taken to be zero. The chart signals, if any of the C_i^+ or C_i^- exceeds a prespecified control limit h. The parameters h is chosen to meet in-control ARL specified by an experimenter. Therefore, ARL is a function of n, h, and k for CUSUM procedure. One can use the C_i^+ to detect an increase in the process dispersion only when corresponding upper one-sided ARL is denoted as ARL^+.

The construction of CUSUM S^2 chart is based on the assumption of normality or at least approximately normality of the process quality characteristic. Amin et al[6] discussed the effect of non-normality on control charts for location. If the process distribution deviates from normal, the ARLs obtained by assuming normality will differ. Table 1 gives the ARL values for CUSUM S^2 chart with sample size $n = 10$, upper control limit $h = 1.5362$ and $k = 1.24$ for the normal, double exponential, uniform, exponential and gamma distributions. Here double exponential is the example of heavy tailed and uniform is the example of light tailed distribution. An effect of skewed distributions on CUSUM S^2 chart is also studied. The upper control limit only considered with various shifts in a standard deviation that is $\sigma = \delta\sigma_0$, where δ be the extent of increase in process standard deviation. Table 1

TABLE 1 ARL^+ performance of CUSUM S^2 chart for n = 10

δ	Normal	Laplace	Uniform	Exponential	Gamma
01	284.2	36.8	448631.0	23.6	37.1
1.2	11.8	9.2	328.6	8.6	9.3
1.4	3.9	4.3	21.2	4.6	4.3
1.6	2.4	2.7	7.9	3.0	2.8
1.8	1.7	2.1	4.8	2.3	2.0
2	1.4	1.7	3.5	1.9	1.7
3	1.0	1.1	1.7	1.2	1.1
4	1.0	1.0	1.3	1.1	1.0
5	1.0	1.0	1.1	1.0	1.0

depicts that if the process distribution is heavy tailed or skewed and control limit is set under normality assumption, then ARLs are very small as compared with the normal. While for light tailed distribution, the ARLs tend to be larger as compared with normal. This implies that for heavy tailed distribution, a false alarm will occur frequently.

Shirke et al[12] developed a sign chart for variability based on in-control deciles, which is a modification of a sign chart based on in-control quartiles given by Amin et al.[6] The chart procedure proposed by Shirke et al[12] is as follows. Consider D_2 and D_8 respectively be the 2^{nd} and 8^{th} deciles, when the process is in-control. We assume that D_2 and D_8 are known from the past data.

Define

$$W_{ij} = \begin{cases} 1 & X_{ij} < D_2 \text{ or } X_{ij} > D_8 \\ 0 & X_{ij} = D_2 \text{ or } X_{ij} = D_8 \\ -1 & D_2 < X_{ij} < D_8, \end{cases} \tag{2}$$

and $W_i = \sum_{j=1}^{n} W_{ij}$. Define a random variable $V_i = (W_i + n)/2$ and has binomial distribution with parameters n and p, where $p = P\ \{X_{ij} < D_2 \text{ or } X_{ij} > D_8 | \sigma = \delta\sigma_0\}$. Moreover, when the process is in-control $p = p_0 = 0.4$. The two-sided chart gives signal if $V_i > c$ or $V_i < n - c$, where c is chosen such that

$$\alpha = \sum_{j=0}^{n-c-1} \binom{n}{j} p_0^j (1-p_0)^{n-j} + \sum_{j=c+1}^{n} \binom{n}{j} p_0^j (1-p_0)^{n-j}. \tag{3}$$

In the upper one-sided case, c is chosen such that

$$\alpha = 1 - \sum_{j=0}^{c} \binom{n}{j} p_0^j (1-p_0)^{n-j}, \tag{4}$$

where α be the false alarm probability, when the process is in-control.

Shirke et al[12] have shown that the chart based on deciles outperforms chart based on quartiles proposed by Amin et al.[6] We extend this approach and provide a nonparametric CUSUM chart to monitor the process dispersion σ^2. We define

$$U_{ij} = \begin{cases} 1 & X_{ij} \le D_2 \text{ or } X_{ij} \ge D_8 \\ 0 & X_{ij} > D_2 \text{ or } X_{ij} < D_8, \end{cases} \tag{5}$$

and $U_i = \sum_{j=1}^{n} U_{ij}$. U_i has binomial distribution with parameters n and p, where $p = P(X_{ij} \le D_2 \text{ or } X_{ij} \ge D_8 | \sigma = \delta\sigma_0)$.

The small shifts in process dispersion can be monitored with the help of the proportion of the observations which falls in the tails. When there is a change in the process variation, we denote p by p_1. Consider $\psi = |p_0 - p_1|$, $\psi > 0$ and we wish to detect a shift of size p_1 quickly. Define a CUSUM monitoring statistic for the i^{th} subgroup sample,

$$\begin{aligned} C_i^+ &= max(0, U_i - (np_0 + k_1) + C_{i-1}^+) \\ C_i^- &= max(0, (np_0 - k_1) - U_i + C_{i-1}^-), \end{aligned} \tag{6}$$

where k_1 is the reference value with $k_1 = \frac{n\psi}{2}$. The initial starting values are mostly chosen as zero, that is, $C_0^+ = 0$ and $C_0^- = 0$. Let H be the parameter of a nonparametric CUSUM chart. If $C_i^+ > H$ or $C_i^- > H$, then the process is thought to be out-of-control. Moreover, $C_i^+ > H$ is used to detect an increase in process dispersion, while $C_i^- > H$ for to detect the decrease in process dispersion. It is noted that

TABLE 2 The (k_1, H) values under $ARL_0^+ \approx 370$

$ARL^+ \approx 370$						
ψ	0.1		0.2		0.3	
n	k_1	H	k_1	H	k_1	H
5	0.25	8.69	0.50	5.00	0.75	3.61
6	0.30	8.11	0.60	5.00	0.90	3.49
7	0.35	8.10	0.70	5.10	1.05	3.50
8	0.40	8.20	0.80	5.00	1.20	3.60
9	0.45	8.80	0.90	5.00	1.35	3.30
10	0.50	9.30	1.00	5.00	1.50	3.51
11	0.55	9.00	1.10	5.00	1.65	3.81
12	0.60	9.98	1.20	5.00	1.80	3.58
13	0.65	9.00	1.30	5.30	1.95	3.69
14	0.70	9.30	1.40	5.00	2.10	3.58
15	0.75	10.20	1.50	5.00	2.25	3.73

TABLE 3 ARL^{+} comparison for various process distributions

	δ	Normal		Laplace		Uniform		Exponential		Gamma	
		N-CUSUM	CUSUM S^2	N-CUSUM	CUSUM S^2	N-CUSUM	CUSUM S^2	N-CUSUM	CUSUM S^2	N-CUSUM	CUSUM S^2
h			1.5362		3.643		0.936		6.957		3.713
ARL*			284.2		36.6		448631.0		23.8		37.0
n=10	1	284.0	284.2	284.0	284.4	284.0	285.5	284.0	283.5	284.0	283.4
H=8.2	1.2	20.2	11.7	28.2	25.1	15.5	16.6	75.3	29.7	56.9	26.1
k=0.5	1.4	9.2	3.9	12.1	8.4	7.6	5.7	29.4	11.5	18.6	8.5
	1.6	6.3	2.3	8.0	4.7	5.5	3.4	16.4	6.9	10.1	4.8
	1.8	5.1	1.7	6.2	3.3	4.5	2.5	11.3	4.9	7.0	3.4
	2	4.4	1.4	5.3	2.6	3.9	2.0	8.8	3.8	5.5	2.6
	3	3.1	1.0	3.5	1.4	2.9	1.2	4.7	1.9	3.2	1.4
	4	2.7	1.0	3.0	1.1	2.6	1.1	3.6	1.4	2.7	1.1
	5	2.5	1.0	2.7	1.0	2.4	1.0	3.2	1.2	2.4	1.0
h			1.1		3.4804		0.39		5.1082		2.887
ARL*			283.6		32.1		605126.4		20.1		32.3
n=15	1	284.4	283.6	284.4	283.0	284.4	285.0	284.4	285.5	284.4	284.5
H=8.8	1.2	15.3	8.2	21.9	16.2	11.6	14.6	64.4	22.9	47.2	18.8
k=0.75	1.4	6.8	2.8	9.0	6.1	5.6	4.2	22.9	8.6	14.0	6.2
	1.6	4.7	1.8	5.9	3.7	4.1	2.3	12.3	5.2	7.5	3.6
	1.8	3.8	1.4	4.6	2.7	3.4	1.7	8.4	3.7	5.2	2.6
	2	3.3	1.2	3.9	2.1	3.0	1.4	6.5	2.9	4.1	2.0
	3	2.4	1.0	2.7	1.2	2.2	1.0	3.5	1.5	2.4	1.2
	4	2.1	1.0	2.3	1.0	2.1	1.0	2.8	1.2	2.1	1.0
	5	2.0	1.0	2.1	1.0	2.0	1.0	2.4	1.1	2.0	1.0
h			0.9		2.3		0.2993		3.495		2.367
ARL*			283.8		29.7		735294.2		17.9		29.3
n=20	1	283.1	283.8	283.1	284.3	283.1	283.8	283.1	283.6	283.1	284.7
H=9.2	1.2	12.4	6.5	17.9	14.3	9.4	12.0	56.0	20.3	40.1	14.8
k=1	1.4	5.5	2.3	7.3	4.8	4.6	3.4	18.8	7.1	11.4	5.0
	1.6	3.8	1.5	4.8	2.9	3.3	1.9	10.0	4.1	6.1	2.9

(Continues)

TABLE 3 (Continued)

δ	Normal		Laplace		Uniform		Exponential		Gamma	
	N-CUSUM	CUSUM S^2	N-CUSUM	CUSUM S^2	N-CUSUM	CUSUM S^2	N-CUSUM	CUSUM S^2	N-CUSUM	CUSUM S^2
1.8	3.1	1.2	3.8	2.1	2.8	1.4	6.8	2.9	4.2	2.1
2	2.7	1.1	3.2	1.7	2.5	1.2	5.3	2.3	3.4	1.7
3	2.0	1.0	2.2	1.1	2.0	1.0	2.9	1.3	2.1	1.1
4	1.9	1.0	2.0	1.0	1.8	1.0	2.3	1.0	1.8	1.0
5	1.8	1.0	1.9	1.0	1.7	1.0	2.1	1.0	1.7	1.0

ARL^* is the in-control ARL when the control limit set under the normality assumption and N-CUSUM is the proposed nonparametric CUSUM chart.

the reference value k_1 and control limit H can be chosen such that they would satisfy the specified ARL.

It is easy to compute ARLs for Shewhart-type control chart and not so for CUSUM and EWMA control charts. There are different methods in the literature to compute ARLs of a CUSUM chart. Brook and Evans[17] have given Markov chain approach to obtain ARL of a CUSUM chart. Yang and Cheng[9] used Markov chain approach to computing ARLs of a CUSUM chart based on sign statistic. We first obtain ARL for the upper one-sided CUSUM chart. We divide the region $(0, H)$ into $M-1$ subintervals of equal width of $2w$, where $w = H/(2(M-1))$. Take 1^{st} subinterval as $(-\infty, 0]$, the k^{th} interval is $(m_k - w, m_k+w)$, where m_k be the midpoints of k^{th} subinterval with $m_1=0$, $m_k=(2k-3)H/(2(M-1))$ for $k=2,3,...,M$; and $(M+1)^{th}$ interval as (H,∞). These all $M+1$ subintervals can be viewed as states of Markov chain. Moreover, the state $M+1$ is the action state, which is absorbing state and remaining M states are transient states of Markov chain $\{C_i^+; i=0,1,\ldots\}$.

Consider the transition probability matrix corresponding to transient states $1,2,\ldots M$ be $R^p = ((p_{kj}^p))$, $(k,j=1,2,...,M)$, whose kj^{th} element represents the transition probability that statistic C_i^+ reaches state j at time i, given that C_{i-1}^+ was in state k at time $(i-1)$. The transition probabilities can be calculated as

$$\begin{aligned} p_{k1}^p &= P(C_i^+ \le 0 | C_{i-1}^+ = m_k) = P(U_i - (np_0 + k_1) \\ &\quad + C_{i-1}^+ \le 0 | C_{i-1}^+ = m_k) \\ &= P(U_i \le np_0 + k - m_k) \\ &= \sum_{s=0}^{[np_0+k_1-m_k]} \binom{n}{s} p^s(1-p)^{n-s}, \end{aligned}$$

$k=1,2,....,M; i=1,2,3,....$

$$\begin{aligned} p_{kj}^p &= P(m_j - w \le C_i^+ < m_j + w | C_{i-1}^+ = m_k) \\ &= P(m_j - w \le U_i - (np_0 + k_1) + C_{i-1}^+ \\ &\qquad < m_j + w | C_{i-1}^+ = m_k) \\ &= P(m_j - m_k - w + np_0 + k_1 \le U_i < m_j - m_k \\ &\qquad + w + np_0 + k_1) \\ &= \sum_{s=0}^{[(m_j-m_k+w+np_0+k_1)^-]} \binom{n}{s} p^s(1-p)^{n-s} \\ &\quad - \sum_{s=0}^{[(m_j-m_k-w+np_0+k_1)^-]} \binom{n}{s} p^s(1-p)^{n-s}, \end{aligned}$$

$k=1,2,....,M, j=2,3,...,M$ and $i=1,2,3,...,$ where $(\boldsymbol{\beta})^-$ be the largest integer not greater than $\boldsymbol{\beta}$. Let b be the $M\times 1$ vector of probabilities that the process started in state $1,2,\ldots M$. In this case $b=(b_1,b_2,...,b_M)'$. Since we considered that $C_0^+ = C_0^- = 0$, we get $b_1=1$ and $b_k=0$

for $k \neq 1$. Consider $P^p = ((p^p_{kj}))$ be a $(M+1)\times(M+1)$ transition probability matrix such that

$$P^p = \left[\begin{array}{c|c} R^p_{M\times M} & P_{M\times 1} \\ \hline 0'_{1\times M} & \mathbf{1} \end{array}\right].$$

Then ARL corresponding to upper one-sided CUSUM chart can be obtained as $ARL^{+}=b'(I-R^p)^{-1}\mathbf{1}'$, where $\mathbf{1}'$ =(1,1,...,1) be the 1×M vector with elements 1. The in-control ARLs can be calculated by substituting $p=p_0$, therefore $ARL^{+} = ARL^{+}_{0}$ be the in-control ARL and if $p=p_1$ then $ARL^{+} = ARL^{+}_{1}$ be the out-of-control ARL. Similar way, one can compute ARL for lower one-sided CUSUM chart, which is denoted by ARL^{-}. Then the ARL for nonparametric CUSUM chart can be calculated as follows:

$$ARL = \frac{1}{1/ARL^{+} + 1/ARL^{-}}. \tag{7}$$

Table 2 gives values of k_1 and H under $\text{ARL}^{+}_{0}\approx 370$ for sample size 5 to 15 and ψ=0.1,0.2,0.3.

3 | PERFORMANCE STUDY OF THE NONPARAMETRIC CUSUM CHART BASED ON DECILES

The performance of control charts can be studied to measure its ability to detect a change in the process parameter quickly. ARL is one of the performance measures, which is used for comparison of control charts. The chart is more efficient, when in-control ARL is large and corresponding out-of-control ARL is small. We have studied the performance of the proposed chart for various process distributions (normal, Laplace, uniform, exponential, and gamma). In the literature, no any standard nonparametric CUSUM chart is available to monitor process dispersion. Therefore, We have compared the performance of proposed nonparametric CUSUM chart with parametric CUSUM S^2 chart.

In most of the situations, early detection of an increase in the process dispersion is of interest and in that case, a one-sided control chart is desirable. The performance of the proposed chart is reported for sample sizes n = 10, 15, 20 with shift δ in process standard deviation. Based on 20,000 runs, the ARL for CUSUM S^2 chart is computed. Table 3 provide ARLs along with various shifts in process standard deviation for Normal $(0,1)$, Laplace $(0,1)$, Uniform $(a = 0, b = \sqrt{12} + a)$, Exponential ($\theta$=1) and Gamma$(a = 2, b = \sqrt{a})$ distribution for sample size $n = 10, 15, 20$. It is clear from Table 3 that an out-of-control ARLs for CUSUM S^2 chart are smaller than nonparametric CUSUM chart, which indicate that CUSUM S^2 chart is more efficient than nonparametric CUSUM chart for all distributions under study. But, such comparison is meaningless because in-control ARL is obtained by using control limit which is set under normality assumption (ARL*). For example, ARL* is quite low (36.6) for Laplace distribution. Suppose we are interested to enhance ARL* from 36.6 to 284 using multiplicative factor (284/36.6=7.75), we get out-of-control ARL to detect shift in variation of 1.2σ as 194.5. This is significantly larger than corresponding out-of-control ARL 28.2 for nonparametric CUSUM chart. Here, we can see that the ARL* of CUSUM S^2 chart changes from 284.2 (for normal distribution) to 36.6, 448631.0, 23.8 and 37, when the process distribution is Laplace, uniform, exponential and gamma

TABLE 4 The ARL^{+}_{1} values under $ARL^{+}_{0}\approx$ 370, ψ=0.1, and p_0=0.4

	p_1					
n	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
9	365.1	17.9	7.1	4.4	3.2	2.6
10	377.9	16.8	6.8	4.3	3.2	2.5
11	367.8	15.6	6.1	3.8	2.8	2.2
12	379.7	14.8	6.1	3.9	2.9	2.2
13	374.7	13.9	5.4	3.4	2.5	2.1
14	367.2	13.0	5.2	3.4	2.5	2.1
15	375.3	12.4	4.9	3.1	2.3	2.0
16	376.0	11.9	4.7	2.9	2.2	2.0
17	364.4	11.3	4.4	2.8	2.1	2.0
18	367.1	10.8	4.4	2.9	2.2	2.0
19	375.3	10.4	4.2	2.7	2.1	1.9

TABLE 5 Piston rings data and values of charting statistic

Sample	X_1	X_2	X_3	X_4	X_5	V	C^+
1	74.030	74.002	74.019	73.992	74.008	3	0.75
2	73.995	73.992	74.001	74.011	74.004	2	0.50
3	73.988	74.024	74.021	74.005	74.002	3	1.50
4	74.002	73.996	73.993	74.015	74.009	1	0.25
5	73.992	74.007	74.015	73.989	74.014	4	3.25
6	74.009	73.994	73.997	73.985	73.993	1	2.00
7	73.995	74.006	73.994	74.000	74.005	0	1.00
8	73.985	74.003	73.993	74.015	73.988	3	4.00
9	74.008	73.995	74.009	74.005	74.004	0	1.75
10	73.998	74.000	73.990	74.007	73.995	1	2.75
11	73.994	73.998	73.994	73.995	73.990	1	2.75
12	74.004	74.000	74.007	74.000	73.996	0	1.75
13	73.983	74.002	73.998	73.997	74.012	2	3.75
14	74.006	73.967	73.994	74.000	73.984	2	3.75
15	74.012	74.014	73.998	73.999	74.007	2	3.75
16	74.000	73.984	74.005	73.998	73.996	1	2.75
17	73.994	74.012	73.986	74.005	74.007	2	3.75
18	74.006	74.010	74.018	74.003	74.000	2	3.75
19	73.984	74.002	74.003	74.005	73.997	1	2.75
20	74.000	74.010	74.013	74.020	74.003	3	4.75
21	73.982	74.001	74.015	74.005	73.996	2	4.50
22	74.004	73.999	73.990	74.006	74.009	1	3.50
23	74.010	73.989	73.990	74.009	74.014	4	6.50
24	74.015	74.008	73.993	74.000	74.010	2	6.25
25	73.982	73.984	73.995	74.017	74.013	4	8.25
26	74.012	74.015	74.030	73.986	74.000	4	10.00
27	73.995	74.010	73.990	74.015	74.001	3	10.75
28	73.987	73.999	73.985	74.000	73.990	3	11.50
29	74.008	74.010	74.003	73.991	74.006	2	11.25
30	74.003	74.000	74.001	73.986	73.997	1	10.25
31	73.994	74.003	74.015	74.020	74.004	2	11.25
32	74.008	74.002	74.018	73.995	74.005	1	10.25
33	74.001	74.004	73.990	73.996	73.998	1	10.25
34	74.015	74.000	74.016	74.025	74.000	3	12.25
35	74.030	74.005	74.000	74.016	74.012	3	13.00
36	74.001	73.990	73.995	74.010	74.024	3	13.75
37	74.015	74.020	74.024	74.005	74.019	4	15.50
38	74.035	74.010	74.012	74.015	74.026	5	18.25
39	74.017	74.013	74.036	74.025	74.026	5	21.00
40	74.010	74.005	74.029	74.000	74.020	3	21.75

respectively for sample size $n=10$. Moreover, ARL* changes from 283.8 to 735294.2 for uniform distribution with $n=20$. It means, there is very low false alarm.

The nonparametric CUSUM chart has smaller out-of-control ARLs when process distributions are uniform and normal. But it has larger ARLs when the process distribution is heavy tailed or skewed like Laplace, exponential and gamma. It is observed that from Table 3 that out-of-control ARLs decreases as sample size increases. In Table 4, one-sided out-of-control ARL values for various values of p_1 and sample size n=9 to 19 are reported. It can be observed that out-of-control ARLs decreases as sample size increases and as tail proportion p_1 increase, that is shift in process dispersion is increases.

4 | EXAMPLE

Here, we illustrate the construction of a nonparametric sign chart based on deciles and proposed CUSUM chart based on deciles with the example inside diameter measurements (mm) for automobile engine piston rings data Montgomery.[18] There are 25 primary samples and 15 additional samples each of size 5, which is described in Table 5. Figures 1 and 2 show that a nonparametric control chart based on in-control deciles with control limit $c=4$ and a nonparametric CUSUM chart with $k_1=0.25$ and $H=8.69$. We can see that a nonparametric control chart based on in-control deciles gives the signal on 38^{th} sample while a CUSUM chart gives the signal on 26^{th} sample.

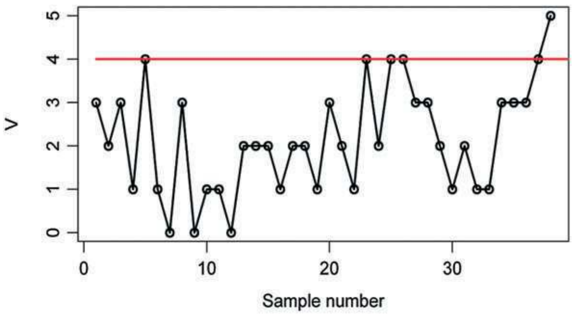

FIGURE 1 A nonparametric control chart based on deciles for piston rings data [Colour figure can be viewed at wileyonlinelibrary.com]

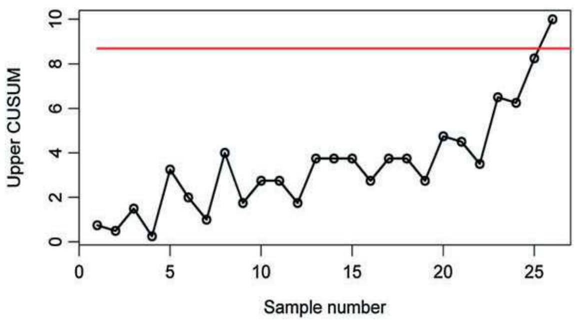

FIGURE 2 A nonparametric CUSUM chart for piston rings data [Colour figure can be viewed at wileyonlinelibrary.com]

5 | CONCLUSION

In the present article, we present a nonparametric CUSUM chart based on in-control deciles for detecting small shifts in process dispersion. Since, whatever be the process distribution the proposed nonparametric CUSUM chart give same in-control ARL. Therefore, the proposed nonparametric chart is a better alternative to CUSUM S^2 chart when process distribution is not known in advance. Moreover, it does not require any distributional assumption. The performance in terms of ARL of the proposed control chart for various distributions is studied. Due to the simplified procedure of proposed CUSUM chart, we recommend for use of proposed CUSUM chart.

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D. T. Shirke is a Professor of Statistics at the Shivaji University, Kolhapur, India. He received his PhD in Statistics from Shivaji University, Kolhapur, India. His research areas include statistical inference and statistical process control. He is an elected member of International Statistical Institute, the Netherlands.

M. S. Barale is PhD student at Department of Statistics, Shivaji University, Kolhapur, India. He received his master's degree in Statistics in 2015 from the Shivaji University, Kolhapur, India.

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